Math 176 Final Practice Version A

1. Explain the following three concepts of calculus. Give a mathematically precise definition while also providing examples with geometric intuition.

(i) Explain the concept of limit.

(ii) Explain the concept of derivative.

(iii) Explain the concept of integral.

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2. Suppose $f(x) = x^2$ and $g(x) = x - 1$. Evaluate the composition

$$(f \circ g)(2) = \boxed{}$$

3. Evaluate the following limits:

$\lim_{x \to e^2} \ln x = \boxed{}$ and $\lim_{x \to 1} \left(\frac{1}{x^2 - x} - \frac{1}{x - 1} \right) = \boxed{}$

4. Under a set of controlled laboratory conditions, the size of the population P of a certain bacteria culture at time t in minutes is described by $P(t) = 3t^3 + 2t + 1$. The rate of population growth at $t = 19$ minutes is

$\boxed{}$ bacteria per minute.

5. Find the following derivatives:

$\frac{d}{dx}(x^3 + 3^x) = \boxed{}$ $\frac{d}{dx}\sqrt{9 + x^2} = \boxed{}$

$\frac{d}{dx}\left(x^2 \ln(3 + x^6)\right) = \boxed{}$

6. The rule for differentiating an inverse funtion is

(A) $\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$

(B) $\frac{d}{dx} f^{-1}(x) = \frac{-1}{f'(f^{-1}(x))}$

(C) $\frac{d}{dx} f^{-1}(x) = \frac{f'(x)}{f^2(x)}$

(D) $\frac{d}{dx} f^{-1}(x) = \frac{-f'(x)}{f^2(x)}$

(E) none of these.

7. Find the absolute maximum and absolute minimum values of $g(x) = 2x^3 - 3x^2 + 1$ on the interval $[0, 2]$.

absolute maximum = $\boxed{}$ absolute minimum = $\boxed{}$

8. The demand function for a certain make of portable hair dryer is given by

$$p = \sqrt{255 - 5x}$$

where p is the unit price in dollars and x is the quantity depanded in hundred units/week. Compute the elasticity of demand $E(p)$, determine whether the demand is elastic, unitary or inelastic and find the consumer surples in dollars/week when the price is set at $p = 10$.

$E(p) =$ _____ $E(10) =$ _____

The demand is

(A) elastic

(B) unitary

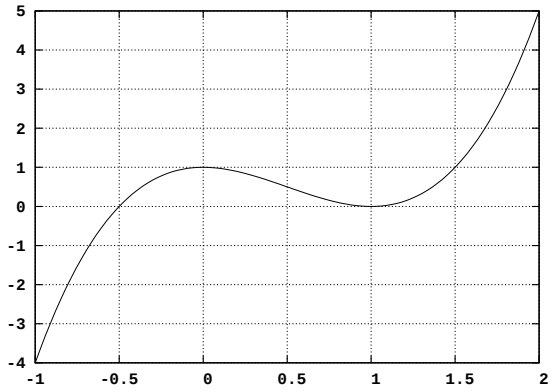
(C) inelastic

Consumer surplus = _____

9. List all critical numbers for the function $f(x) = xe^{-x}$.

$x =$ _____

10. Consider the function $y = f(x)$ given by the following graph:



(True/False) The function has a relative maximum at $x = 0$.

(True/False) The function has an inflection point at $x = 1$.

(True/False) The function is concave down on the interval $[-1, 0.5]$.

11. Use the limit definition of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to explain why the derivative of $f(x) = \sqrt{x}$ is $f'(x) = \dfrac{1}{2\sqrt{x}}$.

12. Explain the product rule $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$ using limits.

13. Find the equation of the line tangent to $x^2y^3 - y^2 + xy = 1$ at the point $(1, 1)$.

14. Find the following antiderivatives:

$\int 5x^2 dx =$ $\qquad$ $\int \ln(2x)\, dx =$ $\qquad$

$\int \frac{x^2}{\sqrt{x^3-1}} dx =$ $\qquad$ $\int e^{2t+3} dt =$ $\qquad$

15. Find the definite integral

$\int_0^4 (x^2 - 3x + 1) dx =$ $\qquad$

16. Sketch the graphs of $f(x) = \sqrt{x}$ and $g(x) = -\frac{1}{2}x - 1$ and find the area of the region enclosed by these graphs and the vertical lines $x = 1$ and $x = 4$.

17. State the Fundamental Theorem of Calculus.