Math 176 Midterm Version C Name: — Key —

1. The equation of the line passing through the points $(1, -2)$ and $(3, 1)$ may be written in the form $ax + by = 1$ where

$a = \boxed{3/7}$ and $b = \boxed{-2/7}$

$m = \frac{1+2}{3-1} = \frac{3}{2}$

$y - 1 = \frac{3}{2}(x-3)$

$\frac{2}{3}y - x = \frac{2}{3} - 3 = \frac{2-9}{3} = \frac{-7}{3}$

$\frac{3}{7}x - \frac{2}{7}y = 1$

2. Suppose $f(x) = x - 2$ and $g(x) = 1/x$. Evaluate the composition

$(f \circ g)(2) = \boxed{-3/2}$

$g(2) = \frac{1}{2}, \; f(\frac{1}{2}) = \frac{1}{2} - 2 = \frac{1-4}{2} = -3/2$

3. Evaluate the following limits:

$\lim_{x \to 2} \sqrt{2 + 3x} = \boxed{2\sqrt{2}}$ and $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \boxed{4}$

$\sqrt{2+6} = \sqrt{8} = 2\sqrt{2}$

$\frac{(x-2)(x+2)}{x-2} = x + 2 \to 2 + 2 = 4$

4. Under a set of controlled laboratory conditions, the size of the population P of a certain bacteria culture at time t in minutes is described by $P(t) = 3t^3 + 2t + 1$. The rate of population growth at $t = 19$ minutes is

$P'(t) = 9t^2 + 2$

$\boxed{3251}$ bacteria per minute.

$19 \times 19 = 171 + 19\_ = 361$; $361 \times 9 = 3249$; $3249 + 2 = 3251$

5. Find the following derivatives:

$\frac{1}{2\sqrt{3+x^4}} \frac{d}{dx}(3+x^4) = \frac{4x^3}{2\sqrt{3+x^4}} = \frac{2x^3}{\sqrt{3+x^4}}$

$\frac{d}{dx}(2x^3 + 5) = \boxed{6x^2}$ $\frac{d}{dx}\sqrt{3 + x^4} = \boxed{2x^3/\sqrt{3+x^4}}$

$\frac{d}{dx}\left[(x+1)^{1/3}(x+3)^{1/4}\right] = \boxed{\frac{1}{3}(x+1)^{-2/3}(x+3)^{1/4} + \frac{1}{4}(x+1)^{1/3}(x+3)^{-3/4}}$

6. The quotient rule is

(A) $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{f(x)^2}$

(B) $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

(C) $\left(\frac{f}{g}\right)'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$

(D) $\left(\frac{f}{g}\right)'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{g(x)^2}$

(E) none of these.

Math 176 Midterm Version C Name: —Key—

7. For the demand equation

$$x = -\frac{5}{4}p + 20$$

compute the elasticity of demand $E(p)$ and determine whether the demand is elastic, unitary or inelastic when $p = 10$.

$E(p) = \boxed{p/(16-p)}$ $E(10) = \boxed{5/3}$

The demand is

(A) elastic

(B) unitary

(C) inelastic

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{p\,5/4}{-\frac{5}{4}p+20} = \frac{p5}{80-5p} = \frac{p}{16-p}$$

$$E(10) = \frac{10}{16-10} = \frac{10}{6} = \frac{5}{3}$$

8. List all critical values for the function $f(x) = x^2/(x+1)$.

$x = \boxed{-2, -1, 0}$

$$f'(x) = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

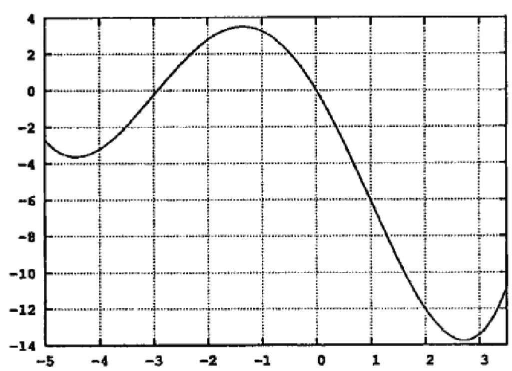
$x(x+2) = 0$

$x = 0, -2$

$(x+1)^2 = 0$

$x = -1$

9. Consider the function $y = f(x)$ given by the following graph:



(True/False) The function has a relative minimum at $x = 0$.

(True/False) The function has an inflection point at $x = -3$.

(True/False) The function is concave down on the interval $[-3, 1]$.

10. Find the absolute maximum and absolute minimum values of $g(x) = -x^2 - 3x + 2$ on the interval $[-3, 2]$.

absolute maximum = $\boxed{17/4}$ absolute minimum = $\boxed{-8}$

$g'(x) = -2x - 3 = 0, \quad x = -3/2$

$g(-3) = -9 + 9 + 2 = 2$

$g(-3/2) = -9/4 + 9/2 + 2 = \frac{-9+18+8}{4} = \frac{17}{4}$

$g(2) = -4 - 6 + 2 = -8$

Math 176 Midterm Version D   Name: — Key —

1. The equation of the line passing through the points $(-1, 5)$ and $(1, 0)$ may be written in the form $ax + by = 1$ where

$a = \boxed{1}$ and $b = \boxed{2/5}$

$m = \frac{5-0}{-1-1} = -\frac{5}{2}$

$y = -\frac{5}{2}(x-1)$

$2y + 5x = 5$

$x + \frac{2}{5}y = 1$

2. Suppose $f(x) = x + 3$ and $g(x) = 1/x$. Evaluate the composition

$$(f \circ g)(2) = \boxed{7/2}$$

$g(2) = \frac{1}{2}$, $f(\frac{1}{2}) = \frac{1}{2} + 3 = 7/2$

3. Evaluate the following limits:

$\lim_{x \to 3} \sqrt{3 + 2x} = \boxed{3}$ and $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \boxed{4}$

$\sqrt{3+6} = \sqrt{9} = 3$

$\frac{(x-2)(x+2)}{x-2} = x+2 \to 2+2 = 4$

4. Under a set of controlled laboratory conditions, the size of the population P of a certain bacteria culture at time t in minutes is described by $P(t) = 3t^3 + 2t + 1$. The rate of population growth at $t = 19$ minutes is

$P'(t) = 9t^2 + 2$

$\boxed{3251}$ bacteria per minute.

$19 \times 19 = 171 + 19\ldots = 361$; $361 \times 9 = 3249$; $3249 + 2 = 3251$

5. Find the following derivatives:

$\frac{1}{2\sqrt{2+x^4}} \frac{d(2+x^4)}{dx} = \frac{4x^3}{2\sqrt{2+x^4}} = \frac{2x^3}{\sqrt{2+x^4}}$

$\frac{d}{dx}(x^4 + 1) = \boxed{4x^3}$ $\frac{d}{dx}\sqrt{2 + x^4} = \boxed{2x^3/\sqrt{2+x^4}}$

$\frac{d}{dx}\left[(x + 2)^{1/3}(x + 1)^{1/4}\right] = \boxed{\frac{1}{3}(x+2)^{-2/3}(x+1)^{1/4} + \frac{1}{4}(x+2)^{1/2}(x+1)^{-3/4}}$

6. The quotient rule is

(A) $\left(\frac{f}{g}\right)'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$

(B) $\left(\frac{f}{g}\right)'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{g(x)^2}$

(C) $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{f(x)^2}$

(D) $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

(E) none of these.

Math 176 Midterm Version D Name: — key —

7. For the demand equation

$$x = -\frac{5}{4}p + 20$$

compute the elasticity of demand $E(p)$ and determine whether the demand is elastic, unitary or inelastic when $p = 10$.

$E(p) = \boxed{p/(16-p)}$ $\qquad E(10) = \boxed{5/3}$

The demand is

(A) elastic

(B) unitary

(C) inelastic

$$E(p) = -\frac{pf'(p)}{f(p)} = \frac{p\,5/4}{-\frac{5}{4}p+20} = \frac{p5}{80-5p} = \frac{p}{16-p}$$

$$E(10) = \frac{10}{16-10} = \frac{10}{6} = \frac{5}{3}$$

8. List all critical values for the function $f(x) = x^2/(x+1)$.

$x = \boxed{-2, -1, 0}$

$$f'(x) = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

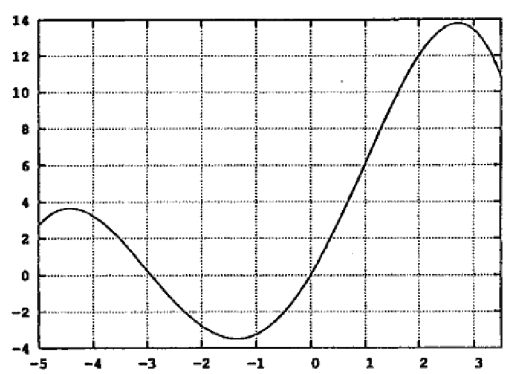
$x(x+2) = 0$

$x = 0, -2$

$(x+1)^2 = 0$

$x = -1$

9. Consider the function $y = f(x)$ given by the following graph:



(True/False) The function has an inflection point at $x = -3$.

(True/False) The function has a relative maximum at $x = 0$.

(True/False) The function is concave down on the interval $[-3, 1]$.

10. Find the absolute maximum and absolute minimum values of $g(x) = -x^2 + 3x + 2$ on the interval $[-2, 3]$.

absolute maximum = $\boxed{17/4}$ $\qquad$ absolute minimum = $\boxed{-8}$

$g(-2) = -4 - 6 + 2 = -8$

$g'(x) = -2x + 3 = 0, \quad x = 3/2$

$g(3/2) = -9/4 + 9/2 + 2 = \frac{-9+18+8}{4} = \frac{17}{4}$

$g(3) = -9 + 9 + 2 = 2$

Math 176 Midterm Version C & D Name: — Key —

11. Use the limit definition of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to explain why the derivative of $f(x) = x^3$ is $f'(x) = 3x^2$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}2}}{\cancel{h}} = \lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2$$

12. Explain the reciprocal rule $(1/g)'(x) = -g'(x)/g(x)^2$ using limits.

$$\left(\frac{1}{g}\right)'(x) = \lim_{h \to 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h} = \lim_{h \to 0} \frac{g(x) - g(x+h)}{h\, g(x+h)\, g(x)}$$

$$= -\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \lim_{h \to 0} \frac{1}{g(x+h)g(x)} = -g'(x) \frac{1}{g(x)^2}$$

13. Find the equation of the line tangent to $y^3 - y = x^2 + xy - 1$ at the point $(1, \sqrt{2})$.

$$3y^2y' - y' = 2x + y + xy'$$

$$(3y^2 - 1 - x)y' = 2x + y$$

$$y' = \frac{2x + y}{3y^2 - 1 - 1}$$

$$m = \frac{2 + \sqrt{2}}{3 \cdot 2 - 1 - 1} = \frac{2 + \sqrt{2}}{4}$$

Eq of line:

$$y - \sqrt{2} = \frac{2 + \sqrt{2}}{4}(x - 1)$$