① Use the limit definition

$$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$$

to explain why $e^x e^y = e^{x+y}$.

$$e^x e^y = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n \lim_{n\to\infty}\left(1+\frac{y}{n}\right)^n = \lim_{n\to\infty}\left(\left(1+\frac{x}{n}\right)\left(1+\frac{y}{n}\right)\right)^n$$

$$= \lim_{n\to\infty}\left(1+\frac{x}{n}+\frac{y}{n}+\cancel{\frac{xy}{n^2}}\right)^n$$

← since n^2 in the denominator is very large, this term is very small compared to the other terms and may be neglected in the limit.

$$= \lim_{n\to\infty}\left(1+\frac{x+y}{n}\right)^n = e^{x+y}$$

② Use the fact that $e^x e^y = e^{x+y}$ to explain why $\ln AB = \ln A + \ln B$.

Let $x = \ln A$ and $y = \ln B$.

Then $e^x = A$ and $e^y = B$.

therefore $AB = e^x e^y = e^{x+y}$ implies

$$\ln(AB) = \ln e^{x+y} = x+y = \ln A + \ln B$$

③ Find the accumulated amount if $P = 150000$ is invested at an interest rate $r = 10\%$ per year for $t = 4$ years compounded monthly.

$$A = P(1+r/n)^{nt} = 150000\left(1+\frac{.1}{12}\right)^{12\cdot 4} \approx 223403.11$$