Final Exam Review

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The Final Exam is on Thursday, December 11 at 12noon–2pm in AB635. Please know well all problems from the Quizzes and Exams. Also study the previous review sheets. Here are some additional practice problems.

1. Quiz I

    a. Especially problems 3, 4 and 5 and variations of these.

    b. Find the sum $\sum_{k=1}^{n}(2k^2 + 3k - 4)$.

    c. Simplify the expression $\ln\left(e^{1-x^2}\right)$.

2. Quiz II

    a. Convert $2.\overline{16}$ into a fraction.

    b. Find the supremum $\sup\{\, x : x^2 < 1 \,\}$.

    c. Let A is a set of real numbers that has a maximum element. Let $\alpha = \max A$ and $B = \{\, 2x : x \in A \,\}$. Find $\sup B$?

3. Quiz III

    a. State the definition of the greatest integer function $[x]$.

    b. Prove that $[2x] = [x] + [x + 1/2]$.

    c. Find the integral $\int_1^8 s(x)dx$ where

$$s(x) = \begin{cases} 1 & \text{for } x \in [0,5] \\ 2 & \text{for } x \in (5,6] \\ 3 & \text{for } x \in (6,10]. \end{cases}$$

4. Quiz IV

    a. Prove equations in exercise 1abcd on page 35 by induction.

    b. Find the vertex of the parabola $y = 2x^2 - 3x + 14$.

5. Quiz V

    a. Compute the integral $\int_2^4 \sqrt{x}\, dx$

b. Compute the integral $\int_{-2}^{2} |(x-1)(x+2)|\, dx$

c. Compute the integral $\int_{5}^{2} 1[2x]\, dx$

6. Quiz VI

  a. Compute the integral $\int_{0}^{\pi/6} \sin(x)\, dx$

7. Exam I

  a. Find a formula for the sum $\sum_{k=3}^{14} \left(\ln(1 + 3\cos x) \right)^{k}$

  b. Compute the integral $\int_{0}^{\pi/12} \sin(3x)\, dx.$

  c. Find the average value of the function $f(x) = 1 + x^3$ on the interval $[1, 4]$.

8. Quiz 8

  a. Find the area of the radial set of $f(\theta) = 2\theta$ over the interval $0 \le \theta \le \pi$.

  b. State the definition in term of δ and ϵ for a function $f(x)$ to be continuous at the point p.

  c. Show that $f(x)$ is continuous at $p = 2$ for $f(x) = x^2$ and $f(x) = 1/x$ and $f(x) = \sqrt{x}$.

9. Quiz 9

  a. Compute the integral $\int_{0}^{\pi/6} (\sin x)^2\, dx$

  b. State the δ-ϵ definition of $\lim_{x \to p} f(x) = A$.

10. Quiz 10

  a. Let f and g be functions such that

  $$\lim_{x \to p} f(x) = A \quad \text{and} \quad \lim_{x \to p} g(x) = B$$

  use the δ-ϵ definition of limit to show that

  $$\lim_{x \to p} (f + g)(x) = A + B \quad \text{and} \quad \lim_{x \to p} (fg)(x) = AB$$

  and if $B \ne 0$ show that
  $$\lim_{x \to p} \left(\frac{f}{g} \right)(x) = \frac{A}{B}.$$

11. Quiz 10

    a. Use the limit laws to compute $\lim\limits_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$

12. Exam 2

    a. Define what it means for a bounded function to be integrable on the interval $[a,b]$.

    b. Define the derivative in terms of limits.

    c. Define the greatest integer function $[x]$.

    d. Define the absolute value $|x|$.

    e. Suppose $g(x)$ is continuous at $x = p$ and $f(x)$ is continuous at $f(p)$. Show that $h(x) = f(g(x))$ is continuous at $x = p$.

    f. Compute the limit $\lim\limits_{h->0} \dfrac{1}{h^2 + 3h} - \dfrac{1}{3h}$.

13. Quiz 11

    a. Find the limit $\lim\limits_{x\to 1} \dfrac{\sin(\sqrt{x} - 1)}{x - 1}$

    b. Prove the product rule using limit laws.

    c. Compute $\dfrac{d}{dx} 2x^3 + 5$

    d. Compute $\dfrac{d}{dx} e^{\cos 2x}$

    e. Compute $\dfrac{d}{dx} 3\ln(1 + x^2)$

14. Quiz 12

    a. Compute $\dfrac{d}{dx} \sin(2x)\cos(3x)$

    b. Compute $\dfrac{d}{dx} \int_0^{\cos x} t^2 dt.$

    c. Prove Rolls theorem

    d. Compute $\dfrac{d}{dx} \ln(1 + e^x).$

    e. Compute $\dfrac{d}{dx} x^{2x}.$

15. More Stuff

    a. Story problems from the handout.

    b. Proof of the First Fundamental Theorem of Calculus.