Math 181 Honors Exam 1 Version A

1. Convert the repeating decimal $3.\overline{14}$ to a fraction.

2. Solve the inequality $x^2 > 4$.

3. Let $w(x) = f(x)g(x)$ where f and g are functions with derivatives f' and g'. Suppose $f(1) = 2$, $g(1) = 2$, $f'(1) = 3$ and $g'(1) = 4$. What is the value of $w'(1)$?

4. Supppose $y = u^2$, $u = 2w + 1$ and $w = x^2$. Find $\dfrac{dy}{dx}$.

5. Find all values of x such that the inequality $|x-5|<1$ holds. Express your answer as an interval.

6. Use the δ-ϵ definition of limit to show that $\lim_{x\to 5}\frac{1}{x}=\frac{1}{5}$.

7. Derive the slope of the line tangent to $f(x)=\sqrt{x}$ at the point $\big(x, f(x)\big)$ where $x>0$ using the method of approxation by secants.

8. Use the δ-ϵ definition of limit to show that

$$\lim_{x\to a} f(x) = L \qquad \text{and} \qquad \lim_{x\to a} g(x) = M$$

implies

$$\lim_{x\to a} \big(f(x)g(x)\big) = LM.$$

9. Use the limits

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \qquad \text{and} \qquad \lim_{h\to 0} \frac{1-\cos h}{h} = 0$$

along with the angle addition formula for sine to show

$$\lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \cos x.$$

10. Use the derivative rules to compute the following derivatives.

(i) $\dfrac{d}{dx}(x^{200} + 4x^2 + 1)$

(ii) $\dfrac{d}{dx}\big((3\sin x) + (4\cos x)\big)$

(iii) $\dfrac{d}{dx}\left(\dfrac{x+1}{x^2+1}\right)$

(iv) $\dfrac{d}{dx}\sqrt{x^2 + 3x + 4}$

11. Do only one of the following:

(i) Let $w(x) = f(x)/g(x)$ where $g(x) \neq 0$. Assuming f and g are continuous functions with derivatives f' and g', show $w'(x) = \big(f'(x)g(x) - f(x)g'(x)\big)/(g(x))^2$ by using the limit laws to compute

$$\lim_{h \to 0} \frac{w(x+h) - w(x)}{h}.$$

(ii) Prove the angle addition formula $\sin(x+y) = (\sin x)(\cos y) + (\cos x)(\sin y)$ using the figure

12. [Extra Credit] Let $w(x) = f(g(x))$ where f and g are continuous functions with derivatives f' and g'. Further assume that $g(x) = g(x+h)$ only when $h = 0$. Use limits to prove the chain rule $w'(x) = f'(g(x))g'(x)$.