Math 181 Honors Quiz 12 Version A

1. Let f be a continuous function. Show that

$$\lim_{h\to 0} \frac{1}{h} \int_t^{t+h} f(x)dx = f(t)$$

using the δ-ϵ defintion of limit.

2. Let f be a continuous function. Prove the Fundamental Theorem of Calculus

$$\frac{d}{dt} \int_a^t f(x)dx = f(t)$$

using the limit definition of derivative and the result of the previous question.

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3. Let f and g be functions such that

$$\lim_{x\to a} f(x) = L \qquad \text{and} \qquad \lim_{x\to a} g(x) = M.$$

Use the δ-ϵ definition of limit to show $\lim_{x\to a} \big(f(x) + g(x)\big) = L + M$.

4. Compute the following definite integrals:

(i) $\displaystyle\int_0^3 (3x^2 - x + 4)\, dx$

(ii) $\displaystyle\int_1^2 x\sqrt{x^2+1}\, dx$