Honors Math 181 Homework 3 Version A

1. Use induction to prove the following statements hold for every natural number n.

   (i) $1 + 3 + 5 + \cdots + (2n + 1) = (n + 1)^2$

   (ii) $1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2 = \dfrac{(n + 1)(2n + 1)(2n + 3)}{3}$

2. Simplify the following sums.

   (i) $\displaystyle\sum_{k=1}^{n} (2k)^3$

   (ii) $\displaystyle\sum_{k=1}^{n} (2k + 1)^3$

3. Use the δ-ϵ definition of continuity to show

   (i) $f(x) = \dfrac{3}{\sqrt{x + 1}}$ is continuous at the point $x_0 = 3$

   (ii) $g(x) = \dfrac{7}{x}$ is continuous at any point $x_0 > 0$

4. Find the following limits.

   (i) $\displaystyle\lim_{n \to \infty} \dfrac{n^2 + 7}{n^2 + 9}$

   (ii) $\displaystyle\lim_{n \to \infty} \left(\sqrt{n^2 + 3n} - \sqrt{n^2 + 7} \right)$

   (iii) $\displaystyle\lim_{n \to \infty} \left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2} + \cdots + \dfrac{n}{n^2} \right)$