Honors Math 181 Homework 4 Version A

1. Solve the following inequalities.

(i) $|x-5| > 3$

(ii) $\left|x + \frac{2}{x}\right| \leq 3$

2. Use induction to verify that

$$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n} \leq 1$$

holds for every natural number $n \geq 3$.

3. Simplify the sum $\displaystyle\sum_{k=n}^{n+5} 2^k$.

4. Find the following limits if they exist. If the limit doesn't exist explain why.

(i) $\displaystyle\lim_{n\to\infty} \frac{17n^2}{n^2+5}$

(ii) $\displaystyle\lim_{n\to\infty} \frac{17n^2}{n+5}$

(iii) $\displaystyle\lim_{n\to\infty} \frac{17n}{n^2+5}$

(iv) $\displaystyle\lim_{n\to\infty} \frac{n}{1+7^n}$

(v) $\displaystyle\lim_{n\to\infty} (2n)^{1/n}$

(vi) $\displaystyle\lim_{n\to\infty} \left(n - \sqrt{n^2+5n}\right)$

(vii) $\displaystyle\lim_{n\to\infty} \left(\frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + \cdots + \frac{9}{10^n}\right)$

(viii) $\displaystyle\lim_{n\to\infty} \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}\right)$

5. Define $h(x) = 3f(x)$. Use the δ-ϵ definition of continuity to show that if $f(x)$ is continuous at x_0 then $h(x)$ is also continuous at x_0.

6. Define $w(x) = f(x) + g(x)$. Use the δ-ϵ definition of continuity to show that if $f(x)$ and $g(x)$ are continuous at x_0 then $w(x)$ is also continuous at x_0.