Honors Math 181 Homework 6 Version A

1. Let $f(x) = x$ and $x_j = a + j\dfrac{b-a}{n}$.

(i) Find $\Delta x_j = x_j - x_{j-1}$.

(ii) Simplify $\displaystyle\sum_{j=1}^{n} f(x_j)\Delta x_j$.

(iii) Compute $\displaystyle\lim_{n\to\infty} \sum_{j=1}^{n} f(x_j)\Delta x_j$.

2. Let $f(x) = \sqrt{x}$ and $x_j = (1 + j/n)^2$.

(i) Find $\Delta x_j = x_j - x_{j-1}$.

(ii) Simplify $\displaystyle\sum_{j=1}^{n} f(x_j)\Delta x_j$.

(iii) Compute $\displaystyle\lim_{n\to\infty} \sum_{j=1}^{n} f(x_j)\Delta x_j$.

(iv) Interpret the above limit as an area and draw a sketch of that area.

3. Use the identities

$$2\sin a \sin b = \cos(a-b) - \cos(a+b)$$
$$2\sin a \cos b = \sin(a-b) + \sin(a+b)$$

and the limits

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1, \qquad \lim_{n\to\infty} \sqrt[n]{p} = 1, \qquad \lim_{n\to\infty} n(\sqrt[n]{p} - 1) = \ln p \qquad \text{for } p > 0$$

along with the other limit laws to find the following limits.

(i) $\displaystyle\lim_{n\to\infty} \left(1 - 2^{1/n}\right)$

(ii) $\displaystyle\lim_{n\to\infty} n\left(1 - 2^{1/n}\right)$

(iii) $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{3x}$

(iv) $\displaystyle\lim_{x\to 0} \frac{\cos(x) - \cos(3x)}{x^2}$