Honors Math 181 Homework 8 Version A

In class we have derived the integral rules

$$\int_a^b \sin x\,dx = -\cos x\Big|_a^b \qquad \int_a^b \cos x\,dx = \sin x\Big|_a^b \qquad \int_a^b \frac{1}{x}\,dx = \ln x\Big|_a^b$$

$$\int_a^b \ln x\,dx = \big(x\ln x - x\big)\Big|_a^b \qquad \int_a^b x^\alpha\,dx = \frac{1}{\alpha+1}x^{\alpha+1}\Big|_a^b \qquad \int_a^b \alpha^x\,dx = \frac{\alpha^x}{\ln\alpha}\Big|_a^b$$

$$\int_a^b \arcsin x\,dx = \big(x\arcsin x + \sqrt{1-x^2}\big)\Big|_a^b$$

$$\int_a^b \arccos x\,dx = \big(x\arccos x - \sqrt{1-x^2}\big)\Big|_a^b$$

and transformation rules

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx \qquad \int_a^b \big(f(x)+g(x)\big)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b f(x/k)\,dx = k\int_{a/k}^{b/k} f(x)\,dx \qquad \int_a^b f(x-k)\,dx = \int_{a-k}^{b-k} f(x)\,dx.$$

1. Use the above rules to find the following integrals:

(i) $\displaystyle\int_0^7 \ln(x+2)\,dx$ **(ii)** $\displaystyle\int_1^4 (x+1)\sqrt{x}\,dx$

(iii) $\displaystyle\int_0^1 3^{2x-1}\,dx$ **(iv)** $\displaystyle\int_0^1 \frac{(x+1)(x+2)}{\sqrt{x+3}}\,dx$

(v) $\displaystyle\int_0^{\pi/8} \cos^2 x\,dx$ **(vi)** $\displaystyle\int_0^{\pi/6} \sin x\cos 2x\,dx$

(vii) $\displaystyle\int_1^7 |1-\ln x|\,dx$

2. Find the following limits:

(i) $\displaystyle\lim_{n\to\infty} n(6^{1/n}-3^{1/n})$

(ii) $\displaystyle\lim_{n\to\infty} \frac{4^{1/n}-1}{2^{1/n}-1}$

(iii) $\displaystyle\lim_{n\to\infty} \left(2+\frac{x}{n}\right)^n$

(iv) $\displaystyle\lim_{n\to\infty} \left(1-\frac{x^2}{3n}\right)^n$

(v) $\displaystyle\lim_{n\to\infty} \sum_{k=3}^{n} \frac{1}{7^k}$