Math 181 Exam 1 Version A

1. Convert the repeating decimal $7.\overline{8}$ into an improper fraction.

2. Solve the inequality $|x - 4| \geq 1$

3. State and prove the Pythagorean theorem.

4. Solve the inequality $|x^2 + 2x - 9| < 6$.

5. Find a formula for each of the following sums.

(i) $\displaystyle\sum_{k=n}^{n+9} k$

(ii) $\displaystyle\sum_{k=1}^{n} (k + 14)^2$

6. Use induction to show that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

for every positive integer n.

7. Solve the limits

(i) $\lim_{n\to\infty}\frac{n+5}{7-2n}$

(ii) $\lim_{n\to\infty}\left(\sqrt{n^2+n}-\sqrt{n^2-3n}\right)$

8. Use ϵ-δ to verify that $f(x) = \dfrac{1}{x}$ is continuous at $x_0 = 4$.

9. Solve the limits

(i) $\displaystyle\lim_{x\to 2} \frac{x^2}{x+3}$.

(ii) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x-1}$.

The limit law

$$\text{if } \lim_{n\to\infty} a_n = L \text{ and } L > 0 \text{ then } \lim_{n\to\infty} \sqrt{a_n} = \sqrt{L}.$$

may be verified by the following argument.

Proof. Suppose $\epsilon > 0$.

Let $\epsilon_1 = \min\left(\frac{L}{2}, \epsilon\big(\sqrt{L/2}+\sqrt{L}\big)\right)$. Then by hypothesis there is N_1 large enough so that $n \ge N_1$ implies $|a_n - L| < \epsilon_1$.

Choose $N = N_1$. Then $n \ge N$ implies $n \ge N_1$ so that

$$|a_n - L| < \frac{L}{2} \quad \text{and} \quad -\frac{L}{2} < a_n - L < \frac{L}{2} \quad \text{and} \quad \frac{L}{2} < a_n < \frac{3L}{2}.$$

Therefore

$$|\sqrt{a_n} - \sqrt{L}| = |\sqrt{a_n} - \sqrt{L}| \cdot \left|\frac{\sqrt{a_n}+\sqrt{L}}{\sqrt{a_n}+\sqrt{L}}\right| = \left|\frac{a_n - L}{\sqrt{a_n}+\sqrt{L}}\right|$$

$$< \frac{\epsilon_1}{|\sqrt{a_n}+\sqrt{L}|} < \frac{\epsilon_1}{\sqrt{L/2}+\sqrt{L}} \le \frac{\epsilon\big(\sqrt{L/2}+\sqrt{L}\big)}{\sqrt{L/2}+\sqrt{L}} = \epsilon.$$

10. Adapt this proof to verify the limit law

$$\text{if } \lim_{x\to 4} f(x) = L \text{ and } L > 0 \text{ then } \lim_{x\to 4} \sqrt{f(x)} = \sqrt{L}.$$