Math 181 Exam 2 Practice Version A

1. State the following integration and differentiation formula:

$\int_a^b \sin x\,dx =$ ____

$\int_a^b \arcsin x\,dx =$ ____ assuming $-1 \le a < b \le 1$

$\int_a^b \cos x\,dx =$ ____

$\int_a^b \arccos x\,dx =$ ____ assuming $-1 \le a < b \le 1$

$\int_a^b x^n\,dx =$ ____

$\int_a^b \sqrt[n]{x}\,dx =$ ____ assuming $0 < a < b$

$\int_a^b \ln x\,dx =$ ____ assuming $0 < a < b$

$\int_a^b \frac{1}{x}\,dx =$ ____ assuming $0 < a < b$

$\int_a^b 5^x\,dx =$ ____

$\frac{d}{dx} \ln x =$ ____ assuming $x > 0$

$\frac{d}{dx} \sin x =$ ____

$\frac{d}{dx} \arcsin x =$ ____ assuming $-1 < x < 1$

$\frac{d}{dx} \cos x =$ ____

$\frac{d}{dx} \arccos x =$ ____ assuming $-1 < x < 1$

$\frac{d}{dx} x^n =$ ____

$\frac{d}{dx} \sqrt[n]{x} =$ ____ assuming $x > 0$

$\frac{d}{dx} e^x =$ ____

$\frac{d}{dx} \frac{1}{x} =$ ____ assuming $x \neq 0$

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2. State in terms of ϵ and δ what it means for $\lim_{x \to x_0} f(x) = L$.

3. State the mean value theorem for integrals.

4. Given a function $f(x)$ state the definition of the derivative $f'(x)$ in terms of limits.

5. Use δ and ϵ to show that $f(x) = x^2$ is continuous at $x_0 = 2$.

6. Find a formula for each of the following sums:

(i) $\displaystyle\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)$

(ii) $\displaystyle\sum_{k=1}^{n}\left(k^2+2^k\right)$

7. Work one of the following:

(i) Let $w \neq x$. Use induction to prove

$$\frac{w^n - x^n}{w - x} = \sum_{k=1}^{n} w^{n-k}x^{k-1} \qquad \text{for} \qquad n = 1, 2, 3, \ldots.$$

(ii) Let $f(x) = \dfrac{1}{x}$. Use the limit definition of derivative to show $f'(x) = -\dfrac{1}{x^2}$.

8. Find the following limits:

(i) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

(ii) $\lim_{x \to 0} \dfrac{\sin 3x}{4 \cos 2x}$

(iii) $\lim_{n \to \infty} \dfrac{1 - \sqrt[n]{3}}{1 - \sqrt[n]{4}}$

(iv) $\lim_{n \to \infty} \left(3 + \dfrac{1}{n}\right)^n$

9. Find the following integrals:

(i) $\displaystyle\int_0^2 \big|(x-1)(x+2)\big|\,dx$

(ii) $\displaystyle\int_2^5 7\sqrt{x-1}\,dx$

(iii) $\displaystyle\int_0^{\pi/6} \cos^2(x/2)\,dx$

(iv) $\displaystyle\int_1^4 \ln\left(x^2+\frac{3}{2}x-1\right)dx$