Math 181 Exam 2 Version B

1. State the following integration and differentiation formula:

$$\int_a^b \cos x\,dx = \boxed{}$$

$$\int_a^b \arccos x\,dx = \boxed{}$$

assuming $-1 \le a < b \le 1$

$$\int_a^b \sin x\,dx = \boxed{}$$

$$\int_a^b \arcsin x\,dx = \boxed{}$$

assuming $-1 \le a < b \le 1$

$$\int_a^b 7^x\,dx = \boxed{}$$

$$\int_a^b \sqrt[n]{x}\,dx = \boxed{}$$

assuming $0 < a < b$

$$\int_a^b \ln x\,dx = \boxed{}$$

assuming $0 < a < b$

$$\int_a^b \frac{1}{x}\,dx = \boxed{}$$

assuming $0 < a < b$

$$\int_a^b x^n\,dx = \boxed{}$$

$$\frac{d}{dx}\ln x = \boxed{}$$

assuming $x > 0$

$$\frac{d}{dx}\cos x = \boxed{}$$

$$\frac{d}{dx}\arccos x = \boxed{}$$

assuming $-1 < x < 1$

$$\frac{d}{dx}\sin x = \boxed{}$$

$$\frac{d}{dx}\arcsin x = \boxed{}$$

assuming $-1 < x < 1$

$$\frac{d}{dx}e^x = \boxed{}$$

$$\frac{d}{dx}\sqrt[n]{x} = \boxed{}$$

assuming $x > 0$

$$\frac{d}{dx}x^n = \boxed{}$$

$$\frac{d}{dx}\frac{1}{x} = \boxed{}$$

assuming $x \neq 0$

2. State in terms of ϵ and N what it means for $\lim_{n\to\infty} a_n = L$.

3. State the mean value theorem for integrals.

4. Given a function $f(x)$ state the definition of the derivative $f'(x)$ in terms of limits.

5. Use δ and ϵ to show that $f(x) = \dfrac{1}{x}$ is continuous at $x_0 = 2$.

6. Find a formula for each of the following sums:

(i) $\sum_{k=1}^{n} \left(\frac{1}{2^k} - \frac{k^2}{2} \right)$

(ii) $\sum_{k=n}^{n^2} (1 + 3k)$

7. Work one of the following:

(i) Use induction to prove

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2 \qquad \text{for} \qquad n = 1, 2, 3, \ldots.$$

(ii) Let $f(x) = x^2$. Use the limit definition of derivative to show $f'(x) = 2x$.

8. Find the following limits:

(i) $\displaystyle\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x}$

(ii) $\displaystyle\lim_{x \to 0} \frac{x \cos 3x}{\sin 2x}$

(iii) $\displaystyle\lim_{n \to \infty} \left(\sqrt[n]{3} - \sqrt[n]{4} \right)$

(iv) $\displaystyle\lim_{n \to \infty} \left(1 + \frac{3}{n} \right)^n$

9. Find the following integrals:

(i) $\displaystyle\int_{-1}^{1} (x^2+1)(x^2-1)\,dx$

(ii) $\displaystyle\int_{0}^{2} \frac{5}{2x+1}\,dx$

(iii) $\displaystyle\int_{0}^{\pi/6} \sin^2(x/2)\,dx$

(iv) $\displaystyle\int_{0}^{2} \left|x^2+\frac{3}{2}x-1\right|dx$