Math 181 Quiz 6 Version A

1. Find a formula for each of the following sums.

(i) $\displaystyle\sum_{k=2}^{n} 5^k$

(ii) $\displaystyle\sum_{k=1}^{n} \left(1+\frac{k}{6}\right)^2$

2. Find the following limits.

(i) $\displaystyle\lim_{x\to 1} \frac{x^2+x-2}{x^2+2x-3}$

(ii) $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{3x}$

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The limit law

$$\text{if } \lim_{x\to x_0} f(x) = L \text{ then } \lim_{x\to x_0} \big(f(x)\big)^2 = L^2$$

may be verified by the following argument.

Proof. Suppose $\epsilon > 0$.

Let $\epsilon_1 = \min\Big(1,\ \epsilon/\big(1+2|L|\big)\Big)$. Then by hypothesis there is $\delta_1 > 0$ small enough so that $0 < |x - x_0| < \delta_1$ implies $|f(x) - L| < \epsilon_1$.

Choose $\delta = \delta_1$. Then $0 < |x - x_0| < \delta$ implies $0 < |x - x_0| < \delta_1$ so that

$$|f(x) + L| = |f(x) - L + 2L| \le |f(x) - L| + 2|L| < \epsilon_1 + 2|L| \le 1 + 2|L|.$$

Therefore

$$\begin{aligned} \big|(f(x))^2 - L^2\big| &= \big|f(x) - L\big| \cdot \big|f(x) + L\big| \\ &< \epsilon_1\big(1 + 2|L|\big) \le \frac{\epsilon}{1 + 2|L|}\big(1 + 2|L|\big) = \epsilon. \end{aligned}$$

3. Adapt this proof to verify that

$$\text{if } \lim_{n\to\infty} a_n = 7 \text{ then } \lim_{n\to\infty} a_n^2 = 49.$$