Math 181 Quiz 1 Version A

**4.** Solve the quadradtic equation $x^2 - x - 6 = 0$.

$$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{25}{4} = 0$$

$$x - \frac{1}{2} = \pm \frac{5}{2}$$

So $x = \frac{1}{2} \pm \frac{5}{2}$.

**5.** Find all values of $x$ which satisfy the inequaltiy $x^2 > \dfrac{1}{x}$.

$$x^2 - \frac{1}{x} > 0$$

$$\frac{x^3 - 1}{x} > 0$$

$$\frac{(x-1)(x^2+x+1)}{x} > 0$$

$$\frac{(x-1)\left(\left(x+\frac{1}{2}\right)^2 - \frac{1}{4} + 1\right)}{x} > 0$$

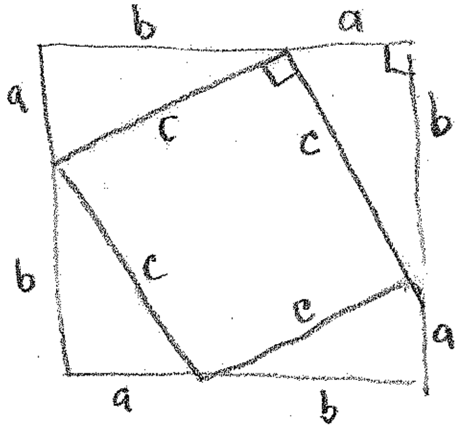
$$\frac{(x-1)\left(\left(x+1/2\right)^2 + \frac{3}{4}\right)}{x} > 0$$

$x = 1$ or $x = 0$

| | | 0 | | 1 | |
|---|---|---|---|---|---|
| $x-1$ | $-$ | | $-$ | | $+$ |
| $x$ | $-$ | | $-$ | | $+$ |
| | $+$ | | $-$ | | $+$ |

solution $(-\infty, 0) \cup (1, \infty)$.

**6.** State and prove the Pythagorean theorem.




Let $a$ and $b$ be the length of the legs of a right triangle and $c$ the length of the hypotenuse. Then

$$a^2 + b^2 = c^2$$

Proof Consider the area of the big square computed in two different ways:

$$A = (a+b)^2 = a^2 + 2ab + b^2$$

$$A = c^2 + 4\left(\tfrac{1}{2}ab\right) = c^2 + 2ab.$$

Thus cancelling $2ab$ we get $a^2 + b^2 = c^2$.

**7.** [Extra credit] In which state is the Courant Institute of Mathematical Sciences?

New York.

Math 181 Quiz 1 Version A

**1.** Answer the following True or False questions:

**(i)** The repeating decimal $1.\overline{9}$ is slightly smaller than 2.

(A) True

(B) False

$1.\overline{9} = 1 + \frac{9}{9} = 2$

**(ii)** For every real value of $x$ it holds that $1 - \sin^2 x = \cos^2 x$.

(A) True

(B) False

$\sin^2 x + \cos^2 x = 1$

by prthagorean theorem

**2.** Convert the repeating decimal $2.\overline{5}$ into an improper fraction.

$$2.\overline{5} = 2 + \frac{5}{9} = \frac{18+5}{9} = \frac{23}{9}$$

**3.** Explain why $\sqrt{xy} \leq \dfrac{x+y}{2}$.

$\boxed{\text{Assume } x, y > 0}$

$$0 \leq (a-b)^2 = a^2 - 2ab + b^2$$

Therefore $\quad 2ab \leq a^2 + b^2$

or $\quad ab \leq \dfrac{a^2+b^2}{2}$

Setting $a = \sqrt{x}$ and $b = \sqrt{y}$ gives

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y} \leq \frac{(\sqrt{x})^2 + (\sqrt{y})^2}{2} = \frac{x+y}{2}$$

(Note if $x, y < 0$ it's not true).