Math 181 Quiz 3 Version A

**1.** Find all values of $x$ which satisfy

**(i)** $x^3 \geq x$

$x^3 - x \geq 0$

$x(x^2-1) \geq 0$

$x(x-1)(x+1) \geq 0$

$x = 0, 1, -1$

| | $-1$ | | $0$ | | $1$ | |
|---|---|---|---|---|---|---|
| $x$ | $-$ | | $-$ | $+$ | | $+$ |
| $x-1$ | $-$ | | $-$ | $-$ | | $+$ |
| $x+1$ | $-$ | | $+$ | $+$ | | $+$ |
| | $-$ | | $+$ | $-$ | | $+$ |

Solution $[-1, 0] \cup [1, \infty)$.

**(ii)** $|x-4| < 1$.

$-1 < x - 4 < 1$

$3 < x < 5$

Solution $(3, 5)$

**2.** Use the $\epsilon$-$\delta$ definition of continuity to show that $f(x) = \dfrac{1}{x}$ is continuous at $x_0 = 4$.

Suppose $\varepsilon > 0$.

Choose $\delta = \min(1, 12\varepsilon)$.

Then $|x-4| < \delta$ implies $-1 < x-4 < 1$ so $3 < x < 5$.

Therefore

$$\left|\frac{1}{x} - \frac{1}{4}\right| = \left|\frac{4-x}{4x}\right| < \frac{\delta}{4|x|} < \frac{\delta}{12} \leq \varepsilon.$$

**3.** Use the summation formulas

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

to find a formula for each of the following sums.

(i) $\displaystyle\sum_{k=3}^{n}(k^2+2k) = \sum_{k=1}^{n} k^2 - 1 - 2^2 + 2\left(\sum_{k=1}^{n} k - 1 - 2\right)$

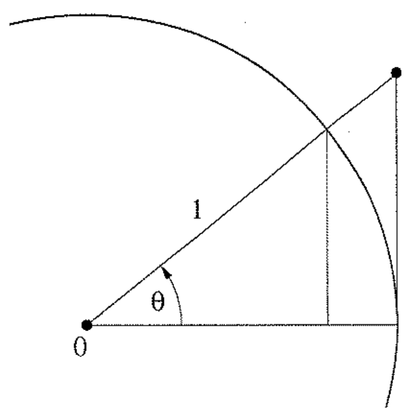
$$= \frac{n(n+1)(2n+1)}{6} - 5 + 2\left(\frac{n(n+1)}{2} - 3\right)$$

$$= \frac{n(n+1)(2n+1)}{6} + n(n+1) - 11.$$

(ii) $\displaystyle\sum_{k=n}^{n+5} k = \sum_{k=1}^{n+5} k - \sum_{k=1}^{n-1} k = \frac{(n+5)(n+6)}{2} - \frac{(n-1)(n)}{2}$

$$= \frac{(n+5)(n+6) - n(n-1)}{2}.$$

**4.** Consider the circle of radius one centered at the origin in the Cartesian plane and the two right triangles depicted below.




$= (1, \tan\theta)$

What are the coordinates of the point $p$ in terms of the angle $\theta$?