Math 181 Honors Exam 1 Review Version A

Topics

1. Field and Order Axioms
2. Solving Inequalities
3. Set Notation and Operations on Sets
4. Completing the Square
5. Graphing and Transformations of Graphs
6. Finding Domains of Real Valued Functions
7. Proof of Pythagorean Theorem
8. Limit of a Repeating Decimal
9. Limit of Continued Fractions
10. Mathematical Definition of Limit
11. Verifying Limits Using the Definition
12. Definition of Continuity
13. Limit Laws
14. Verifying Continuity Using the Definition

Example Problems

1. Field and Order Axioms

(i) State the Field Axioms.

(ii) State the Order Axioms.

(iii) Use the Order Axioms

(POS1) If a, b are positive, so is ab and $a + b$.

(POS2) If a is a number, then either a is positive, or $a = 0$, or $-a$ is positive, and these possibilities are mutually exclusive.

to show that $a < b$ and $b < c$ implies $a < c$.

2. Solving Inequalities

(i) Find all $x \in \mathbf{R}$ such that $(x-2)(x+4)(x-4) > 0$.

(ii) Find all $x \in \mathbf{R}$ such that $x^2 - 3x + 2 < 0$.

(iii) Find all $x \in \mathbf{R}$ such that $\left|x + \dfrac{1}{x}\right| \geq 6$.

(iv) Find all $x \in \mathbf{R}$ such that $\left|\dfrac{1}{x-5}\right| \geq \dfrac{2}{x-7}$.

3. Set Notation and Operations on Sets

(i) Find $A \cup B$, $A \cap B$ and $A \setminus B$ where $A = \{\, 1, 2, 3 \,\}$ and $B = \{\, 2, 4, 8 \,\}$.

(ii) Find $A \cup B$, $A \cap B$ and $A \setminus B$ where $A = [1, 4]$ and $B = [0, 3]$.

4. Completing the Square

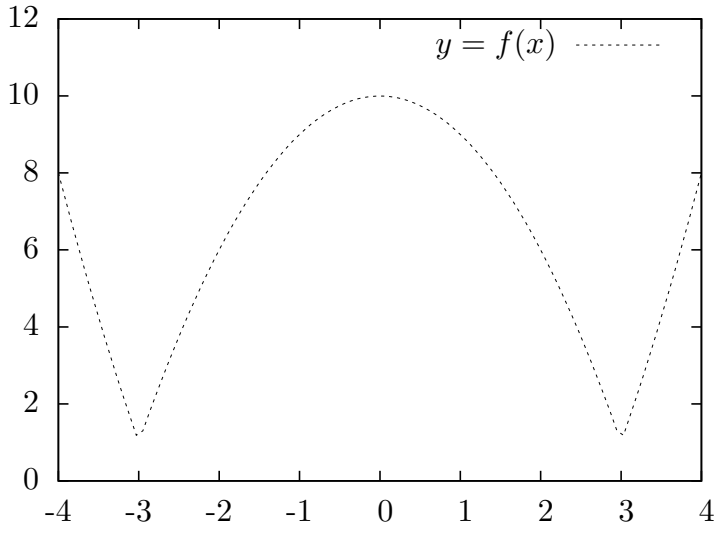
(i) Find the vertex of the parabola $y = x^2 + 3x - 8$.

(ii) Find the center of the hyperbola $x^2 + 3x - 2y^2 + 7y = 100$.

5. Graphing and Transformations of Graphs

(i) Sketch the graph of $y = |x - 3| + 1$.

(ii) Sketch the graphs of $y = f(x+2)$ and $y = -f(x)+3$ where $y = f(x)$ is given by the graph



6. Finding Domains of Real Valued Functions

(i) Find the domain of $f(x) = \dfrac{1}{\sqrt{x^2 - 9}}$.

(ii) Find the domain of $g(x) = \dfrac{1}{x^2 + 9}$.

(iii) Find the domain of $h(x) = \dfrac{1}{|x| - 3}$.

7. Proof of Pythagorean Theorem

(i) State the hypothesis and conclusion and then prove the Pythagorean theorem.

8. Limit of a Repeating Decimal

(i) Write the repeating decimal $0.0\overline{1}$ as a fraction.

(ii) Write the repeating decimal $0.0\overline{10}$ as a fraction.

(iii) Write the repeating decimal $0.0\overline{101}$ as a fraction.

9. Limit of Continued Fractions

(i) Write the continued fraction $[1, \overline{3}]$ in the form $\dfrac{a + \sqrt{b}}{c}$.

(ii) Write the continued fraction $[\overline{2, 1}]$ in the form $\dfrac{a + \sqrt{b}}{c}$.

10. Mathematical Definition of Limit

(i) State the meaning of $\lim_{x\to a} f(x) = L$ in terms of ϵ and δ.

11. Verifying Limits Using the Definition

(i) Use the ϵ-δ definition to verify $\lim_{x\to 2} 3x = 6$.

(ii) Use the ϵ-δ definition to verify $\lim_{x\to 9} \dfrac{1}{\sqrt{x}} = \dfrac{1}{3}$.

(iii) Use the ϵ-δ definition to verify $\lim_{x\to 3} \dfrac{1}{2x-5} = 1$.

(iv) Use the ϵ-δ definition to verify $\lim_{x\to 4} x\sqrt{x} = 8$.

12. Definition of Continuity

(i) State what it means for a function f to be continuous at a in terms of limits.

13. Limit Laws

(i) The 6 limit laws are

(0) $\lim_{x\to a} c = c$

(1) $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$

(2) $\lim_{x\to a} \big(f(x) + g(x)\big) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(3) $\lim_{x\to a} \big(f(x)g(x)\big) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$

(4) $\lim_{x\to a} \dfrac{1}{f(x)} = \dfrac{1}{\lim_{x\to a} f(x)}$ provided $\lim_{x\to a} f(x) \neq 0$

(5) $\lim_{x\to a} f\big(g(x)\big) = f\Big(\lim_{x\to a} g(x)\Big)$ if f is continuous at $\lim_{x\to a} g(x)$.

Use the ϵ-δ definition to verify each limit law.

(ii) Explain how limit law (1) is a special case of limit law (3).

(iii) Explain how limit law (4) is a special case of limit law (5).

(iv) Use the limit laws to show

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

provided $\lim_{x\to a} g(x) \neq 0$.

14. Verifying Continuity Using the Definition

(i) Use the limit laws and the fact that $\lim_{x\to 2} x = 2$ to show that $f(x) = 1/x$ is continuous at the point $x = 2$.

(ii) Use the limit laws and the fact that $\lim_{x\to a} x = a$ to show that $g(x) = x^2$ is continuous at the point $x = a$.

(iii) Use geometry to show that $h(x) = \sin x$ is continuous at $x = 0$.