Math 181 Honors Exam 1 Version A

1. The Order Axioms are

(POS1) If a, b are positive, so is ab and $a+b$.
(POS2) If a is a number, then either a is positive, or $a = 0$, or $-a$ is positive, and these possibilities are mutually exclusive.

Use the order axioms to show to show that $a > b$ and $b > c$ implies $a > c$.

2. Find all $x \in \mathbf{R}$ such that $\dfrac{1}{x-2} > x$.

3. Write the repeating decimal $1.3\overline{7}$ as a fraction.

4. Suppose $A = [-1, 7]$ and $B = (2, 3]$.

(i) Find $A \cup B$.

(ii) Find $A \cap B$.

(iii) Find $A \setminus B$.

5. Find the vertex of the parabola $y = 2x^2 + 5x - 1$.

6. Sketch the graph of $y = |x + 1| - 3$.

7. Find the domain of the real valued function given by $f(x) = \sqrt{|x + 1| - 3}$.

8. State the hypothesis and conclusion and then prove the Pythagorean theorem.

9. Write the continued fraction $[1, \overline{4}]$ in the form $\dfrac{a+\sqrt{b}}{c}$.

10. State the meaning of $\lim_{x\to a} f(x) = L$ in terms of ϵ and δ.

11. Use the ϵ-δ definition to verify $\lim_{x\to 2} x^2 = 4$.

12. The 6 limit laws are

(0) $\lim\limits_{x\to a} c = c$

(1) $\lim\limits_{x\to a} cf(x) = c\lim\limits_{x\to a} f(x)$

(2) $\lim\limits_{x\to a} \big(f(x)+g(x)\big) = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$

(3) $\lim\limits_{x\to a} \big(f(x)g(x)\big) = \lim\limits_{x\to a} f(x) \lim\limits_{x\to a} g(x)$

(4) $\lim\limits_{x\to a} \dfrac{1}{f(x)} = \dfrac{1}{\lim\limits_{x\to a} f(x)}$ provided $\lim_{x\to a} f(x) \neq 0$

(5) $\lim\limits_{x\to a} f\big(g(x)\big) = f\Big(\lim\limits_{x\to a} g(x)\Big)$ if f is continuous at $\lim\limits_{x\to a} g(x)$.

(i) Use the ϵ-δ definition to verify limit law 2.

(ii) Use the limit laws and the fact that $\lim\limits_{x\to 2} x = 2$ to show $f(x) = \dfrac{1}{x+1}$ is continuous at the point $x = 2$.

13. [Extra Credit] Use geometry to show $\lim_{x \to 0} \sin x = 0$