Math 181 Honors Final Review Version A

1. Convert the repeating decimals to fractions:

(i) $3.4\overline{2}$

(ii) $0.\overline{27}$

2. Solve the inequality $\dfrac{x^2-5}{x-1} \geq 0$.

3. The Order Axioms are

(POS1) If a, b are positive, so is ab and $a+b$.

(POS2) If a is a number, then either a is positive, or $a=0$, or $-a$ is positive, and these possibilities are mutually exclusive.

Use the order axioms to show to show that $a > b$ and $b > c$ implies $a > c$.

4. Suppose $A = [0, 3]$ and $B = [2, 7)$.

(i) Find $A \cup B$.

(ii) Find $A \cap B$.

(iii) Find $A \setminus B$.

5. Find the vertex of the parabola $y = 3x^2 + 9x + 2$.

6. Sketch the graph of $y = |x - 2| - 5$.

7. Find the domain of the real valued function given by $f(x) = \sqrt{|x-2| - 5}$.

8. State what it means for $f(x)$ to be continuous at the point $x = a$.

9. Write the continued fraction $[2,\overline{3}]$ in the form $\dfrac{a+\sqrt{b}}{c}$.

10. State the meaning of $\lim_{x\to a} f(x) = L$ in terms of ϵ and δ.

11. Use the ϵ-δ definition to verify $\lim_{x\to 1} \dfrac{1}{x} = 1$.

12. The 6 limit laws are

(0) $\lim\limits_{x\to a} c = c$

(1) $\lim\limits_{x\to a} cf(x) = c\lim\limits_{x\to a} f(x)$

(2) $\lim\limits_{x\to a} \big(f(x)+g(x)\big) = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$

(3) $\lim\limits_{x\to a} \big(f(x)g(x)\big) = \lim\limits_{x\to a} f(x)\lim\limits_{x\to a} g(x)$

(4) $\lim\limits_{x\to a} \dfrac{1}{f(x)} = \dfrac{1}{\lim\limits_{x\to a} f(x)}$ provided $\lim_{x\to a} f(x) \neq 0$

(5) $\lim\limits_{x\to a} f\big(g(x)\big) = f\Big(\lim\limits_{x\to a} g(x)\Big)$ if f is continuous at $\lim\limits_{x\to a} g(x)$.

(i) Use the ϵ-δ definition to verify limit law 3.

13. Suppose $0 < \theta < \pi/2$. Use geometry to show $\cos\theta \leq \dfrac{\sin\theta}{\theta} \leq \dfrac{1}{\cos\theta}$.

14. Suppose $x > 0$ and $h > 0$. Use geometry to show $\dfrac{h}{x+h} \leq \ln(x+h) - \ln x \leq \dfrac{h}{x}$.

15. Use the limit laws to find the following limits.

(i) $\lim_{x\to\infty} x^2 - x$

(ii) $\lim_{x\to -1} \frac{x^2-4}{x^2+1}$

(iii) $\lim_{x\to 2^+} \frac{x-2}{x^2-x-2}$

(iv) $\lim_{x\to 2^+} \frac{x}{2-x}$

16. Fill in the derivatives in the following table:

$\dfrac{d}{dx}x^r =$		$\dfrac{d}{dx}\dfrac{1}{x^r} =$	
$\dfrac{d}{dx}\sin x =$		$\dfrac{d}{dx}\arcsin x =$	
$\dfrac{d}{dx}\cos x =$		$\dfrac{d}{dx}\arccos x =$	
$\dfrac{d}{dx}\tan x =$		$\dfrac{d}{dx}\arctan x =$	

17. State the definition of derivative in terms of limits.

18. Suppose $f(x) = 3x^2$. Use the limit laws to verify $f'(x) = 6x$.

19. Use Calculus to find the following derivatives.

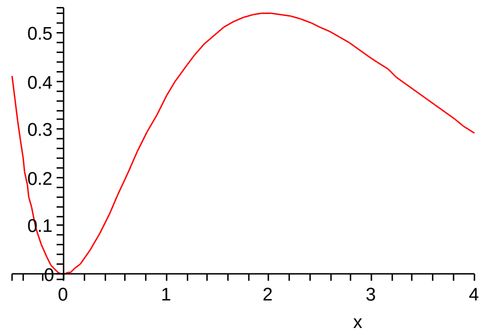
(i) $\dfrac{d}{dx}\left(3x^2 - x + 16\right)$

(ii) $\dfrac{d}{dx}\dfrac{x}{3 + \arctan x}$

(iii) $\dfrac{d}{dx}(x \cos x)$

(iv) $\dfrac{d}{dx} \ln\left(\dfrac{1}{1 + x^2}\right)$

20. Consider the function $f(x) = x^2 e^{-x}$ graphed below



(i) Find the critical points of $f(x)$ on the interval $[-0.5, 4]$.

(ii) Find the maximum value of $f(x)$ on the interval $[-0.5, 4]$.

(iii) Find the minimum value of $f(x)$ on the interval $[-0.5, 4]$.

21. Let $f\colon \mathbf{R} \to \mathbf{R}$ be a continuous function and define

$$g(x) = \int_0^x f(t)\, dt.$$

Use geometry and the δ-ϵ definition of limit to show that $g'(x) = f(x)$.