Section 1.2

① $\boxed{x \in (-3,3)}$

④ $|x-5| < |x+1| \Rightarrow x-5 < x+1$ or $x-5 > -x-1$

$\Rightarrow -5 < 1$ or $2x > 4$

$\Rightarrow$ no solutions or $x > 2$

$\boxed{x \in (2,\infty)}$

⑤ $(x+1)(x-2) < 0 \Rightarrow$ case 1: $x+1 > 0$, and $x-2 < 0$

or case 2: $x+1 < 0$, and $x-2 > 0$

Case 1:

$x+1 > 0 \Rightarrow x > -1$
$x-2 < 0 \Rightarrow x < 2$ $\Big\}$ $-1 < x < 2 = x \in (-1,2)$

Case 2:

$x+1 < 0 \Rightarrow x < -1$
$x-2 > 0 \Rightarrow x > 2$ $\Big\}$ contradiction

$\boxed{x \in (-1,2)}$

⑨ $x^2(x-1) \geq 0 \Rightarrow$ as $x^2 \geq 0$, it must be the case that $x-1 \geq 0$

$x-1 \geq 0 \Rightarrow x \geq 1$ or $x^2 = 0 \Rightarrow x = 0$

$\boxed{x \in [1,\infty) \cup \{0\}}$

⑬ $(4x+7)^{20}(2x+8) < 0 \Rightarrow$ note that $(4x+7)^{20} = [(4x+7)^{10}]^2 > 0$ (theorem 1)

thus $2x+8 < 0 \Rightarrow 2x < -8$

$\Rightarrow x < -4$

$\boxed{x \in (-\infty,-4)}$

(14) Note that $x = x + y - y$. Then

$$|x| = |x + y - y| \underbrace{\leq}_{\text{theorem } 3} |x+y| + |-y| = |x+y| + |y|.$$

Subtracting $|y|$ from each side of the inequality, we have

$$|x| - |y| \leq |x+y|,$$

which is the desired result. ■

(16) By the triangle inequality, we have

$$|x + (-y)| \leq |x| + |-y|.$$

$|x + (-y)| = |x-y|$ and $|-y| = y$, thus

$$|x-y| \leq |x| + |y|.$$ ■

Section 2.3

(1) $f(\frac{3}{4}) = \frac{1}{3/4} = \boxed{\frac{4}{3}}$

$f(-\frac{2}{3}) = \frac{1}{-2/3} = \boxed{-\frac{3}{2}}$

(2) $f(2x+1) = \boxed{\frac{1}{2x+1}}$, $x \neq -\frac{1}{2}$

(5) $f(x)$ is defined when $x^2 - 2 \neq 0 \Rightarrow x^2 \neq 2$

$\Rightarrow \boxed{x \neq \pm\sqrt{2}}$

$f(5) = \frac{1}{5^2-2} = \frac{1}{25-2} = \boxed{\frac{1}{23}}$

(6) $f(x)$ is defined for $\boxed{\text{all } x \in R}$

$f(27) = \sqrt[3]{27} = \boxed{3}$

(7) $f(1) = \frac{1}{|1|} = \boxed{1}$ $\qquad f(-3) = \frac{-3}{|-3|} = \boxed{-1}$

$f(2) = \frac{2}{|2|} = \boxed{1}$ $\qquad f(-\frac{4}{3}) = \frac{-4/3}{|-4/3|} = \boxed{-1}$

(8) $f(1) = 1 + |1| = \boxed{2}$ $\qquad f(-4) = -4 + |4| = \boxed{0}$

$f(\frac{1}{2}) = \frac{1}{2} + |\frac{1}{2}| = \boxed{1}$ $\qquad f(-5) = -5 + |-5| = \boxed{0}$

Section 1.4

(1) $a^x = 2^3 = \boxed{8}$

$x^a = 3^2 = \boxed{9}$

(6) $a^x = 3^2 = \boxed{9}$

$x^a = 2^3 = \boxed{8}$