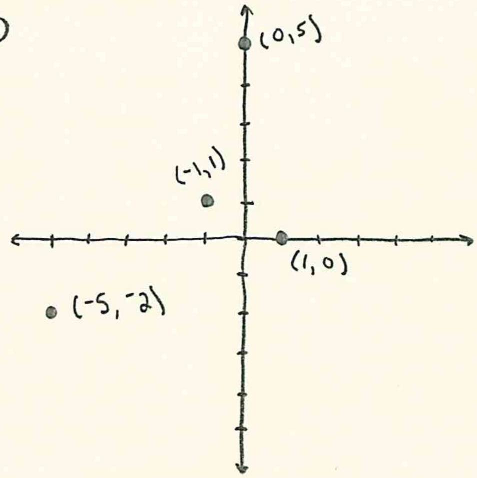
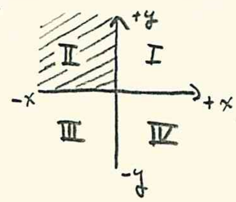
Lang, Section 2.1

①



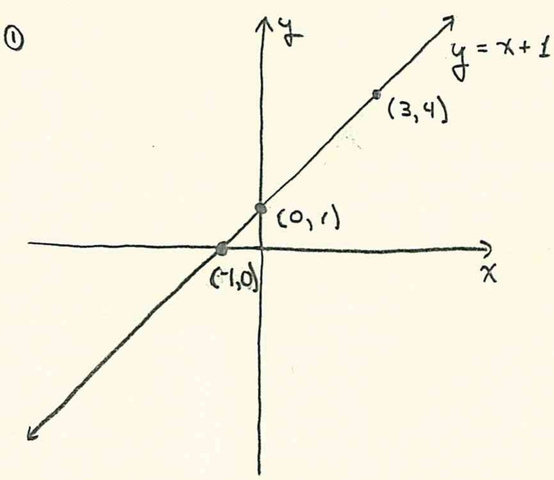
③ If (x,y) is in the second quadrant, then x is negative and y is positive:



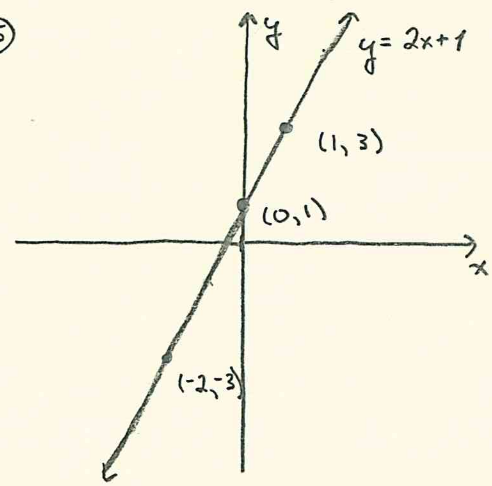
④ If (x,y) is in the third quadrant, then x and y are both negative. See the picture in ③.

Lang, Section 2.2

①

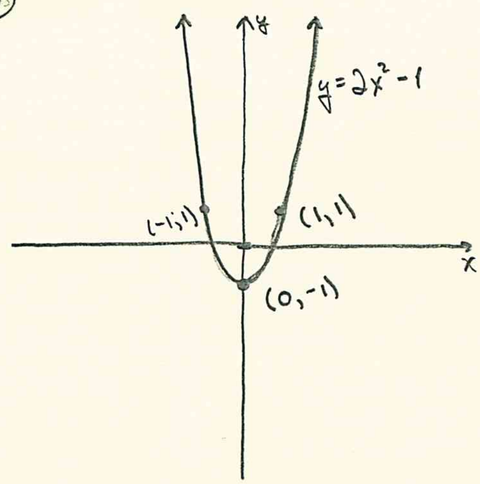


⑤

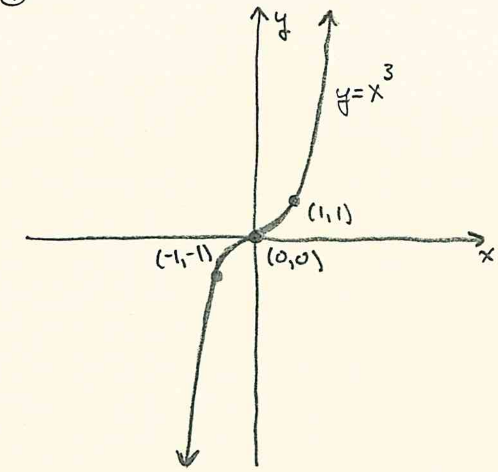


HOMEWORK #2

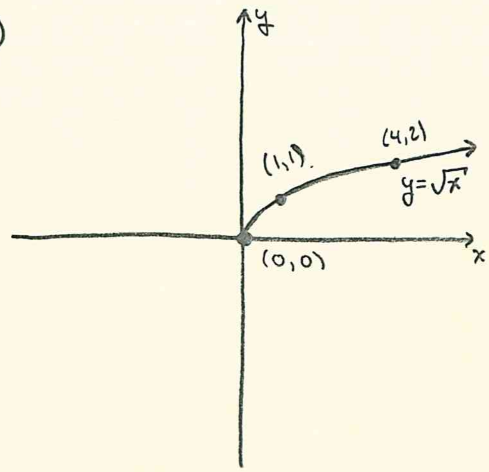
⑨



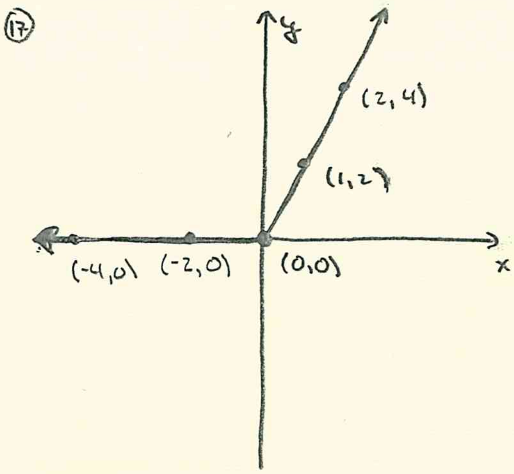
⑪



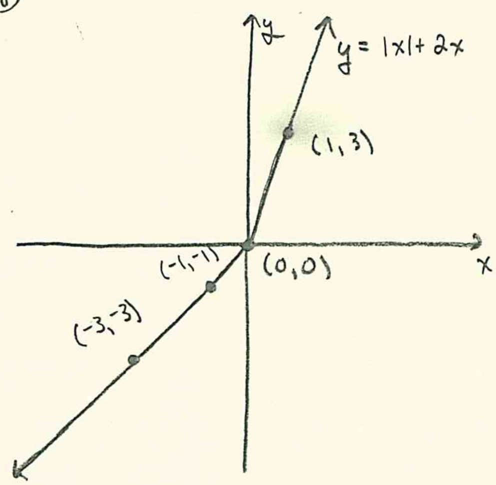
⑬



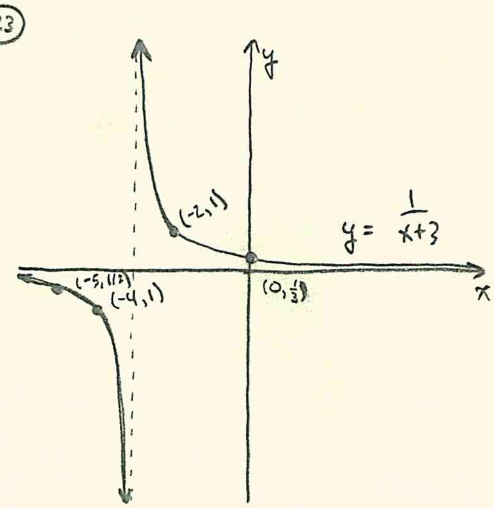
⑰



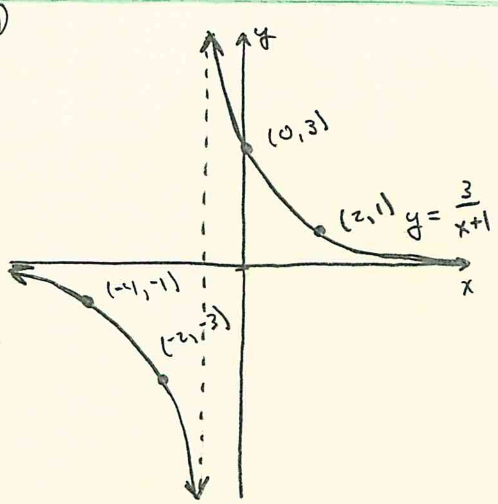
⑱



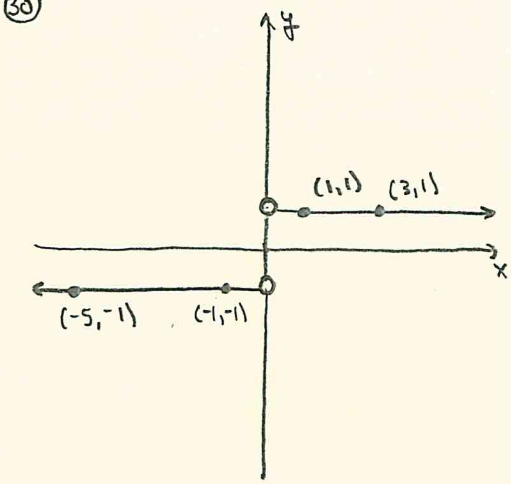
㉓



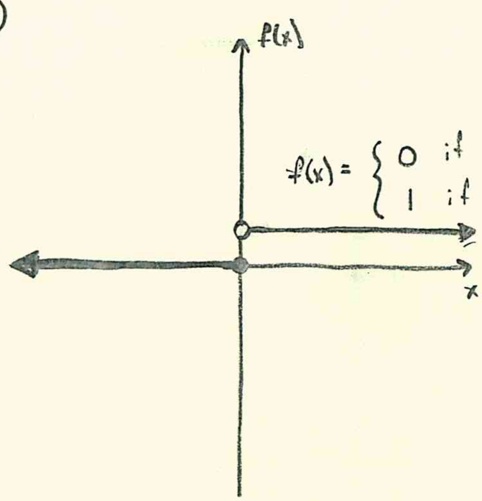
(29)



(30)

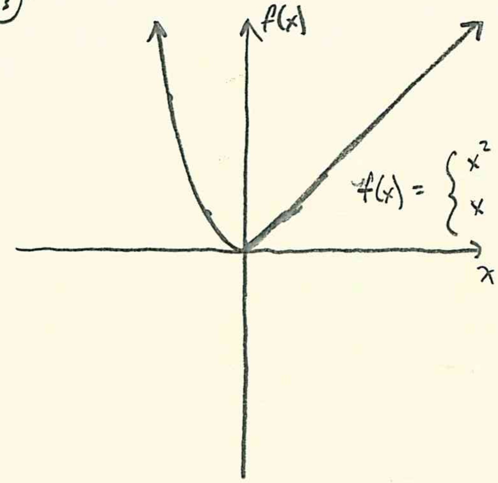


(31)



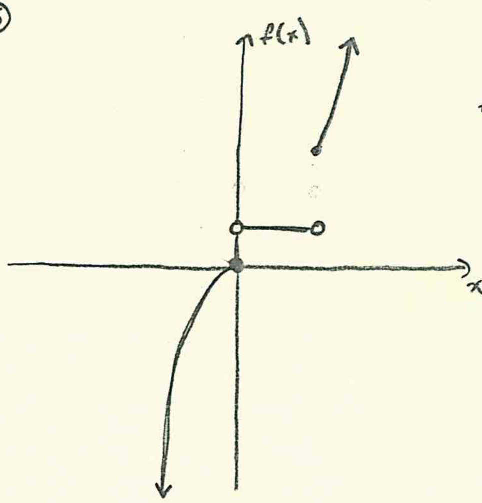
$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(33)



$$f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ x & \text{if } x \ge 0 \end{cases}$$

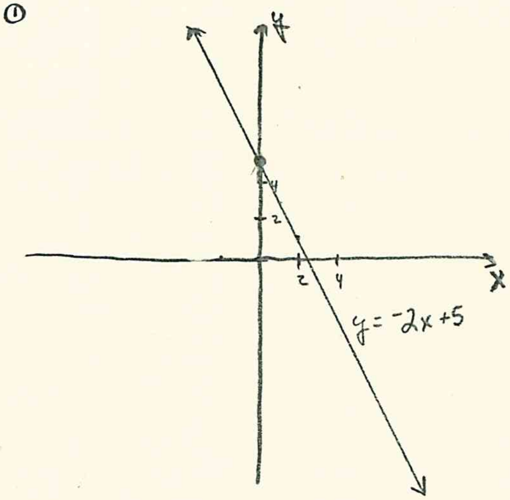
(35)



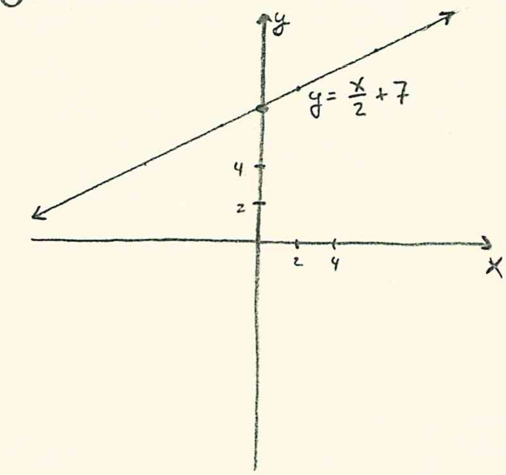
$$f(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ 1 & \text{if } 1 < x < 2 \\ x^2 & \text{if } x \ge 2 \end{cases}$$

Lang, Section 2.3

①



③



⑤ slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-1}{2--1} = \frac{-8}{3} = -\frac{8}{3}$

point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\frac{8}{3}(x - (-1))$$
$$= -\frac{8}{3}(x + 1)$$
$$\boxed{y = -\frac{8}{3}x - \frac{5}{3}}$$

⑥ slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - \frac{1}{2}}{4 - 3} = \frac{-\frac{3}{2}}{1} = -\frac{3}{2}$

point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y + 1 = -\frac{3}{2}(x - 4)$$
$$= -\frac{3}{2}x + 6$$
$$\boxed{y = -\frac{3}{2}x + 5}$$

⑨ $y - y_1 = m(x - 1) \Rightarrow y - 1 = 4(x - 1)$

$$= 4x - 4$$
$$\boxed{y = 4x - 3}$$

A & M, Chapter 1

(14) (a) $\dfrac{3x-1}{2x+3} > 3$

if $2x+3>0 \Rightarrow 2x > -3 \Rightarrow x > -\frac{3}{2}$

then $3x-1 > 3(2x+3) = 6x+9 \Rightarrow -3x > 10 \Rightarrow x < -10/3$

contradiction

if $2x+3 < 0 \Rightarrow 2x < -3 \Rightarrow x < -3/2$

then $3x-1 < 3(2x+3) = 6x+9 \Rightarrow -3x < 10 \Rightarrow x > -10/3$

so $x \in (-10/3, -3/2)$.

(e) $|2x-1/x| > 2 \Rightarrow 2x-1/x > 2$ or $2x-1/x < -2$

if $x>0$, $\cancel{2x-1 > 2x}$ or $2x-1 < -2x \Rightarrow 4x < 1 \Rightarrow x < \frac{1}{4}$ $\boxed{x \in (0, \frac{1}{4})}$

if $x<0$, $2x-1 < 2x$ or $2x-1 > -2x \Rightarrow$ always true,

as $2x-1 < 2x \Rightarrow -1 < 0$ $\boxed{x \in (-\infty, 0)}$

A & M, Chapter 2

(17) $(2,4)$ is the midpoint of the line joining (x,y) and $(1,5)$. Thus

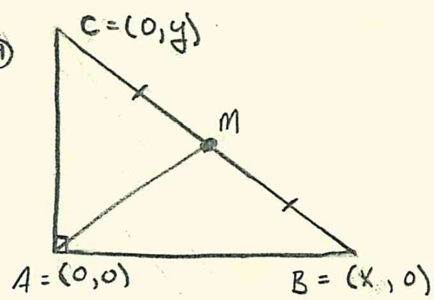
$$2 = \frac{x+1}{2} \quad \text{and} \quad 4 = \frac{y+5}{2}.$$

Solving for x and y, we have

$$4 = x+1 \qquad 8 = y+5$$
$$x = 3 \qquad y = 3$$

Therefore $(x,y) = (3,3)$.

(19)



It follows from the midpoint formula that $M = (\frac{1}{2}x, \frac{1}{2}y)$. As M is the midpoint of $\overline{BC}$, we know that $MB = MC$. It remains to show that $MA = MB$. By the distance formula

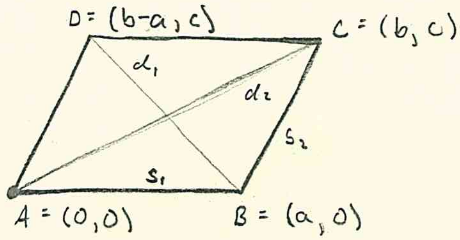
$$MA = \sqrt{(\tfrac{1}{2}x - 0)^2 + (\tfrac{1}{2}y - 0)^2} = \sqrt{\tfrac{1}{4}x^2 + \tfrac{1}{4}y^2}.$$

Similarly,

$$MB = \sqrt{(x - \tfrac{1}{2}x)^2 + (0 - \tfrac{1}{2}y)^2} = \sqrt{\tfrac{1}{4}x^2 + \tfrac{1}{4}y^2}.$$

Therefore $MA = MB = MC$. ∎

21



We want to show that $2s_1^2 + 2s_2^2 = d_1^2 + d_2^2$. Applying the distance formula, we have

$$\begin{aligned} 2s_1 + 2s_2 &= 2\left((a-0)^2 + (0-0)^2\right) + 2\left((b-a)^2 + (c-0)^2\right) \\ &= 2a^2 + 2(b^2 - 2ab + a^2 + c^2) \\ &= 4a^2 + 2b^2 + 2c^2 - 4ab. \qquad (*) \end{aligned}$$

Now applying the distance formula to the diagonals, we have

$$\begin{aligned} d_1^2 + d_2^2 &= \left(((b-a)-a)^2 + (c-0)^2\right) + \left((b-0)^2 + (c-0)^2\right) \\ &= (b-2a)^2 + c^2 + b^2 + c^2 \\ &= b^2 - 4ab + 4a^2 + c^2 + b^2 + c^2 \\ &= 4a^2 + 2b^2 + 2c^2 - 4ab. \qquad (**) \end{aligned}$$

The expressions at (*) and (**) are identical, which proves the desired identity. ∎