Math 181 Honors Homework 5 Worksheet Version A

1. Solve the following inequalities.

(i) $|x+1| < 2x$

(ii) $|x^2 - 5| \geq 1$

(iii) $x + \dfrac{1}{x} \leq 7$

2. Use the ϵ-δ definition of limit to verify the limits.

(i) $\lim\limits_{x\to 1} \dfrac{x}{x+1} = \dfrac{1}{2}$

(ii) $\lim\limits_{x\to -3} \dfrac{1}{x^2} = \dfrac{1}{9}$

(iii) $\lim\limits_{x\to 2} x^3 = 8$

3. Suppose

$$\lim_{x\to 4} f(x) = 5 \qquad \text{and} \qquad \lim_{x\to 4} g(x) = -2.$$

Use the ϵ-δ definition of limit verify the limits.

(i) $\lim\limits_{x\to 4} 3f(x) = 15$

(ii) $\lim\limits_{x\to 4} \big(x + g(x)\big) = 2$

(iii) $\lim\limits_{x\to 1} f(4x) = 5$

4. Consider the following sequence of approximations given by

$$x_1 = 2, \qquad x_2 = 2 + \sqrt{2}, \qquad x_3 = 2 + \sqrt{2 + \sqrt{2}},$$

$$x_4 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}},$$

$$x_5 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$$

and the limit

$$L = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$$

(i) Use your calculator to find decimal representations of x_2, x_3, x_4 and x_5. Write your answer with at least 7 digits of accuracy.

(ii) Use a technique similar to the one developed in class for continued fractions to find the limit L.