A & M, Chapter 7:

(9) Let $\varepsilon > 0$ and choose $\delta < \min\{1, \varepsilon/8\}$. Let $x \in \mathbb{R} \setminus \{2\}$ such that $|x - 2| < \delta$. As $\delta < 1$, we have

$$-1 < x - 2 < 1 \quad \Rightarrow \quad 6 < x + 5 < 8$$
$$\Rightarrow \quad |x + 5| < 8. \qquad (*)$$

Then

$$\begin{aligned} |(x^2 + 3x) - 10| &= |(x-2)(x+5)| \\ &= \underbrace{|x - 2|}_{<\delta \text{ by hypothesis}} \underbrace{|x + 5|}_{< 8 \text{ by } (*)} \\ &< 8\delta \\ &< 8 \cdot \varepsilon/8 \\ &= \varepsilon. \end{aligned}$$

■

(11) (a) Let $\varepsilon > 0$. By hypothesis, $\lim_{x \to a} f(x) = A$, so there exists a $\delta_1$ such that if $x \in \mathcal{D}(f)$ and $|x - a| < \delta_1$, then $|f(x) - A| < \varepsilon/2$. Similarly, as $\lim_{x \to a} g(x) = B$, then is a $\delta_2$ such that if $x \in \mathcal{D}(g)$ and $|x - a| < \delta_2$, then $|g(x) - B| < \varepsilon/2$.

Choose $\delta < \min\{\delta_1, \delta_2\}$. Then if $x \in \mathcal{D}(f) \cap \mathcal{D}(g)$ and $|x - a| < \delta$, we have

$$\begin{aligned} |(f(x) + g(x)) - (A + B)| &= |(f(x) - A) + (g(x) - B)| \\ &\leq |f(x) - A| + |g(x) - B| \\ &< \varepsilon/2 + \varepsilon/2 \\ &= \varepsilon. \end{aligned}$$

■

16 (c) $\lim_{x\to 1} \frac{(3x-1)^2}{(x+1)^3} = \frac{(3\cdot(\lim_{x\to 1} x) - 1)^2}{((\lim_{x\to 1} x) + 1)^3}$

$= \frac{(3\cdot 1 - 1)^2}{(1+1)^2}$

$= \frac{2^2}{2^3}$

$= \boxed{\frac{1}{2}}$

(e) $\lim_{x\to 2} \frac{x-2}{\sqrt{x^2-4}} = \lim_{x\to 2} \frac{x-2}{\sqrt{(x-2)(x+2)}}$

$= \lim_{x\to 2} \frac{(x-2)\sqrt{x-2}}{\sqrt{x+2}\,\sqrt{x-2}}$

$= \lim_{x\to 2} \frac{x-2}{\sqrt{x+2}}$

$= \boxed{0}$

(f) $\lim_{x\to 2} \frac{\sqrt{x-2}}{x^2-4} = \lim_{x\to 2} \frac{\sqrt{x-2}}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{\sqrt{x-2}}{(\sqrt{x-2})^2(x+2)}$

$= \lim_{x\to 2} \frac{1}{\sqrt{x-2}\,(x+2)}$

$= \frac{1}{0} \longrightarrow$ the limit does not exist

(d) $\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{x-2}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{1}{x+2}$

$= \boxed{\frac{1}{4}}$

16 b

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{(h^3 + 3h^2x + 3hx^2 + x^3) - x^3}{h}$$

$$= \lim_{h\to 0} \frac{h(h^2 + 3hx + 3x^2)}{h}$$

$$= \lim_{h\to 0} h^2 + 3hx + 3x^2$$

$$= \boxed{3x^2}$$

17 d

$$\lim_{x\to +\infty} \frac{-2x^3+7}{5x^2-3x-4} = \lim_{h\to 0^+} \frac{-2(\frac{1}{h})^3 + 7}{5(1/h)^2 - 3(1/h) - 4}$$

$$= \lim_{h\to 0^+} \frac{(-2+7h^3)/h^3}{(5-3h-4h^2)/h^2}$$

$$= \lim_{h\to 0^+} \frac{7h^3 - 2}{(-4h^2-3h+5)h}$$

$$= \lim_{h\to 0^+} \frac{7h^3-2}{-4h^3-3h^2+5h}$$

$$= \frac{0-2}{-0-0+0} \rightarrow$$ limit does not exist – diverges to $\boxed{-\infty}$

17 e

$$\lim_{x\to\infty} (3x^3 - 25x^2 - 12x - 17)$$

$$= \lim_{h\to 0^-} \frac{3}{h^3} - \frac{25}{h^2} - \frac{12}{h} - \frac{17}{1}$$

$$= \lim_{h\to 0^-} \frac{3 - 25h - 12h^2 - 17h^3}{h^3}$$

$$= \frac{3}{\lim_{h\to 0^-} h^3}$$

$$= \frac{3}{-0} \rightarrow$$ limit does not exist – diverges to $\boxed{-\infty}$

18 Oct HOMEWORK #6

(18) (a) $\lim_{x \to +\infty} \frac{2x+3}{4x-5} = \lim_{h \to 0^+} \frac{2/h + 3}{4/h - 5}$

$= \lim_{h \to 0^+} \frac{(2+3h)/h}{(4-5h)/h}$

$= \lim_{h \to 0^+} \frac{2+3h}{4-5h}$

$= \frac{2}{4} = \boxed{\frac{1}{2}}$

(b) $\lim_{x \to +\infty} \frac{2x^2+1}{6+x-3x^2} = \lim_{h \to 0^+} \frac{2/h^2+1}{6+1/h-3/h^2}$

$= \lim_{h \to 0^+} \frac{(2+h^2)/h^2}{(6h^2+h-3)/h^2}$

$= \lim_{h \to 0^+} \frac{2+h^2}{-3+h+6h^2}$

$= \boxed{-\frac{2}{3}}$

(c) $\lim_{x \to +\infty} \frac{x}{x^2+5} = \lim_{h \to 0^+} \frac{1/h}{(1+5h^2)/h^2}$

$= \lim_{h \to 0^+} \frac{h}{5h^2+1}$

$= \frac{0}{1} = \boxed{0}$

(d) $\lim_{x \to +\infty} \frac{x^2+5x+6}{x+1} = \lim_{h \to 0^+} \frac{(1+5h+6h^2)/h^2}{(1+h)/h}$

$= \lim_{h \to 0^+} \frac{1+5h+6h^2}{h+h^2}$

$= \frac{1}{0^+} \longrightarrow$ limit does not exist – diverges to $\boxed{+\infty}$

Lang §3.3

② $f(x) = \dfrac{1}{2x+1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1/(2(x+h)+1) - 1/(2x+1)}{h}$$

$$= \lim_{h\to 0} \frac{(2x+1)-(2(x+h)+1)}{h(2(x+h)+1)(2x+1)}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(2x+\underbrace{2h}_{\to 0}+1)(2x+1)}$$

$$= \boxed{-\frac{2}{(2x+1)^2}}$$

③ $f(x) = \dfrac{x}{x+1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)/(x+h+1) - x/(x+1)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)(x+1) - x(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{x^2+x+hx+h-x^2-hx-x}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(x+\underbrace{h}_{\to 0}+1)(x+1)}$$

$$= \boxed{\frac{1}{(x+1)^2}}$$

⑧ $f(x) = x^5$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^5 - x^5}{h}$$

$$= \lim_{h\to 0} \frac{x^5+5x^4h+10x^3h^2+10x^2h^3+5xh^4+h^5-x^5}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5x^4+\cancel{10x^3h}^{\to 0}+\cancel{10x^2h^2}^{\to 0}+\cancel{5xh^3}^{\to 0}+\cancel{h^4}^{\to 0})}{\cancel{h}}$$

$$= \boxed{5x^4}$$