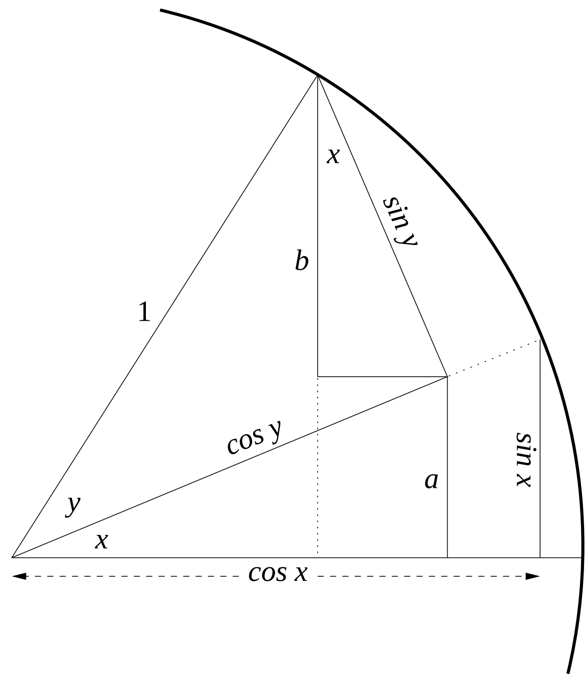
Angle Addition Formula **Math 181 Honor Calculus**

Theorem. For all real values x and y we have

$$\sin(x+y) = \sin x \cos y + \cos x \sin y.$$

Proof: Consider the following diagram.



By similar triangles

$$\frac{a}{\cos y} = \frac{\sin x}{1} \qquad \text{so that} \qquad a = \sin x \cos y$$

and also

$$\frac{b}{\sin y} = \frac{\cos x}{1} \qquad \text{so that} \qquad b = \cos x \sin y.$$

Therefore

$$\sin(x+y) = a + b = \sin x \cos y + \cos x \sin y.$$

Remarks: The difficult part of the proof is drawing the diagram. To check that you understand the diagram and the proof, please complete the following study activities:

1. Place a square to indicate each right angle in the diagram.
2. In the computation of b there is a fraction with the number 1 in it's denominator. Find the line in the diagram corresponding to the 1 that appears in this fraction.
3. Explain why the two angles marked x in the diagram are equal.
4. Redo the proof from memory using the letters a and b for the angles.
5. This proof assumes $x > 0$ and $y > 0$ and $x + y < \pi/2$. Draw another diagram where $0 < x < \pi/2$ and $0 < y < \pi/2$ and $x + y > \pi/2$, then redo the proof for this case.