Math 181 Honors Exam 2 Version B

1. Use the rules of Calculus to find the following derivatives:

(i) $\dfrac{d}{dx}\dfrac{x}{2+\cos x}$

(ii) $\dfrac{d}{dx}(5^x x^3)$

(iii) $\dfrac{d}{dx}(1+|x|)^x$

(iv) $\dfrac{d}{dx}\arctan(\arctan x)$

2. State the definition of the limit

$$\lim_{x\to a} f(x) = L$$

in terms of δ and ϵ.

3. State the definition of the derivative $f'(x)$ in terms of limits.

4. Show that if $f'(x)$ exists at c then $f(x)$ is continuous at c.

5. Suppose $f(x) = \sqrt{x}$. Use the limit definition of derivative to show $f'(x) = 1/(2\sqrt{x})$.

6. Suppose $w(x) = 1/f(x)$ where $f(x)$ is differentiable. Use the limit definition of derivative to show $w'(x) = -f'(x)/\big(f(x)\big)^2$.

7. Consider the function $f(x) = (1 - x)e^x$.

(i) Find $f'(x)$.

(ii) Find the unique ξ such that $f'(\xi) = 0$.

(iii) Show $f(x)$ is increasing on $(-\infty, \xi)$ and decreasing on (ξ, ∞).

(iv) Show that $e^x - 1 \leq xe^x$ for every $x \in \mathbf{R}$.

8. Prove one of the following results:

(i) Linear Approximation Theorem. Let f be twice continuously differentiable on an interval containing a and b. Then there is a point c between a and b such that

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(c)}{2}(b-a)^2.$$

(ii) Generalized Mean Value Theorem. Suppose f and g are differentiable on (a,b) and continuous on $[a,b]$. If $g'(x) \neq 0$ in (a,b), then there exists a point c in (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}.$$

9. [Extra Credit] Give a proof the other theorem in question 8 that you didn't already prove on the previous page.