Math 181 Final Review Version A

Name: ______________________ **Recitation:** ______________________

This answer sheet is the only page you will turn in. Please remove it from the rest of the test and record your answers in the spaces provided.

1.	$\lim_{x \to a} f(x) = L$ means for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $0 < \lvert x-a \rvert < \delta$, then $\lvert f(x) - L \rvert < \varepsilon$.
2.	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
3.	$y' = -\frac{y\cos x + \sin y}{x\cos y + \sin x}$
4.	$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{k=0}^{n-1} f\left(a + k\frac{b-a}{n}\right) \frac{b-a}{n}$

5(i).	3	5(ii).	$1/2$	5(iii).	1
6(i).	$\frac{2}{1+4x^2}$	6(ii).	$\frac{7-x^2}{(7+x^2)^2}$	6(iii).	$3\lvert x\rvert^{3x}(1+\ln\lvert x\rvert)$
7(i).	$\frac{5}{4}x^4 - \frac{2}{3}x^3 + C$	7(ii).	$\frac{1}{3}\sin(x^3+1) + C$	7(iii).	$\frac{2}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$
8(i).	$7/3$	8(ii).	1	8(iii).	$\ln 2$
9(i).	132 cm^2/sec	9(ii).	$\frac{20}{3}$ ft/sec	9(iii).	$30\sqrt[3]{162} \approx 163.54$ \$
10(i).	(T) (F)	10(ii).	(T) (F)	10(iii).	(T) (F)

Math 181 Final Review Version A

1. Precisely define $\lim_{x\to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Define the derivative $f'(x)$ of a function $f(x)$ using limits.

3. Suppose $x\sin y + y\sin x = 3$. Find dy/dx by implicit differentiation.

4. Define the integral $\int_a^b f(x)dx$ of a function $f(x)$ using limits.

5. Find the following limits:

(i) $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x-2}$

(ii) $\lim_{x\to\infty} \dfrac{x^2 + x - 3}{2x^2 - 4}$

(iii) $\lim_{t\to 0} \dfrac{1-e^{-t}}{t}$

6. Find the following derivatives:

(i) $\dfrac{d}{dx}\arctan(2x)$

(ii) $\dfrac{d}{dx}\left(\dfrac{x}{x^2+7}\right)$

(iii) $\dfrac{d}{dx}|x|^{3x}$

7. Find the following antiderivatives:

(i) $\int (5x^3 - 2x^2)\,dx$

(ii) $\int x^2\cos(x^3+1)\,dx$

(iii) $\int x\sqrt{x+1}\,dx$

8. Compute the following areas:

(i) $\int_1^2 x^2 dx$

(ii) $\int_0^{\pi/2} \sin 2x\,dx$

(iii) $\int_1^3 \dfrac{1}{x+1}\,dx$

9. Solve the following story problems:

(i) The length of a rectangle is increasing at a rate of 7 cm/s and its width is increasing at a rate of 6 cm/s. When the length is 15 cm and the width is 6 cm, how fast is the area of the rectangle increasing?

(ii) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 4 ft/s along a straight path. How fast is the tip of his shadow moving when he is 35 ft from the pole?

(iii) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

10. Answer the following true/false questions:

(i) If f is differentiable at a, then f is continuous at a.

(ii) If f is continuous on $[a, b]$, then the integral $\int_a^b f(x)dx$ exists.

(iii) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.

#1. The $\lim_{x\to a} f(x) = L$ means for every $\varepsilon > 0$ there is $\delta > 0$ such that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

#2. $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

#3 $$x \sin y + y \sin x = 3$$

$$\sin y + x\cos y\, y' + y' \sin x + y\cos x = 0$$

$$(x\cos y + \sin x)\, y' = -y\cos x - \sin y$$

$$y' = -\frac{y\cos x + \sin y}{x\cos y + \sin x}$$

#4 Here are three equivalent definitions, any of which would constitute a correct answer.

(i) $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=0}^{n-1} f\left(a + k\frac{b-a}{n}\right)\frac{b-a}{n}$$

(ii) $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(a + k\frac{b-a}{n}\right)\frac{b-a}{n}$$

(iii) Let $x_k = a + k\frac{b-a}{n}$ and $t_k \in [x_{k-1}, x_k]$ for each k. Then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(t_k)\frac{b-a}{n}.$$

5i. $\lim_{x \to 2} \frac{x^2-x-2}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$

$= \lim_{x \to 2} (x+1) = 2+1 = 3$

5ii $\lim_{x \to \infty} \frac{x^2+x-3}{2x^2-4} = \lim_{x \to \infty} \frac{1+\frac{1}{x}-\frac{3}{x^2}}{2-\frac{4}{x^2}}$

$= \frac{1+0-0}{2-0} = \frac{1}{2}$

5iii. $\lim_{t \to 0} \frac{1-e^{-t}}{t} = \lim_{t \to 0} \frac{\frac{d}{dt}(1-e^{-t})}{\frac{d}{dt}t}$

$= \lim_{t \to 0} \frac{0+e^{-t}}{1} = e^0 = 1$

6i $\frac{d}{dx} \arctan(2x) = \frac{1}{1+(2x)^2} \frac{d}{dx} 2x$

$= \frac{1}{1+4x^2} \cdot 2 = \frac{2}{1+4x^2}$

6ii $\frac{d}{dx}\left(\frac{x}{x^2+7}\right) = \frac{1\cdot(x^2+7) - x(2x+0)}{(x^2+7)^2}$

$= \frac{7-x^2}{(7+x^2)^2}$

6iii $\frac{d}{dx}|x|^{3x} = \frac{d}{dx} e^{3x\ln|x|} = e^{3x\ln|x|} \frac{d}{dx}(3x\ln|x|)$

$= |x|^{3x}\left(3\ln|x| + 3x\cdot\frac{1}{x}\right) = 3|x|^{3x}(1+\ln|x|)$

7i $\int (5x^3 - 2x^2)\,dx = 5 \cdot \frac{1}{4}x^4 - 2 \cdot \frac{1}{3}x^3 + C$

$= \frac{5}{4}x^4 - \frac{2}{3}x^3 + C$

7ii $\int x^2 \cos(x^3+1)\,dx$

$u = x^3+1 \quad \frac{du}{dx} = 3x^2, \quad dx = \frac{du}{3x^2}$

$= \int \cancel{x^2} \cos u \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int \cos u\,du$

$= \frac{1}{3}\sin u + C = \frac{1}{3}\sin(x^3+1) + C$

7iii $\int x\sqrt{x+1}\,dx$

$u = x+1, \quad \frac{du}{dx} = 1, \quad du = dx$

$= \int x\sqrt{u}\,du = \int (u-1)\sqrt{u}\,du$

$= \int (u^{3/2} - u^{1/2})\,du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

8iii $\int_1^2 x^2\,dx = \frac{1}{3}x^3\Big|_1^2 = \frac{1}{3}2^3 - \frac{1}{3}1^3$

$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

8ii $\int_0^{\pi/2} \sin 2x \, dx$

$u = 2x \qquad \frac{du}{dx} = 2 \qquad dx = \frac{du}{2}$

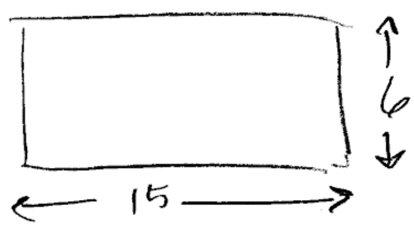
$= \int_0^{\pi} \sin u \, \frac{du}{2} = -\frac{\cos u}{2}\Big|_0^{\pi}$

$= -\frac{\cos \pi}{2} + \frac{\cos 0}{2} = \frac{-(-1)}{2} + \frac{1}{2} = 1$

8iii $\int_1^3 \frac{1}{x+1} \, dx = \ln(x+1)\Big|_1^3$

$= \ln 4 - \ln 2 = \ln \frac{4}{2} = \ln 2$

9i



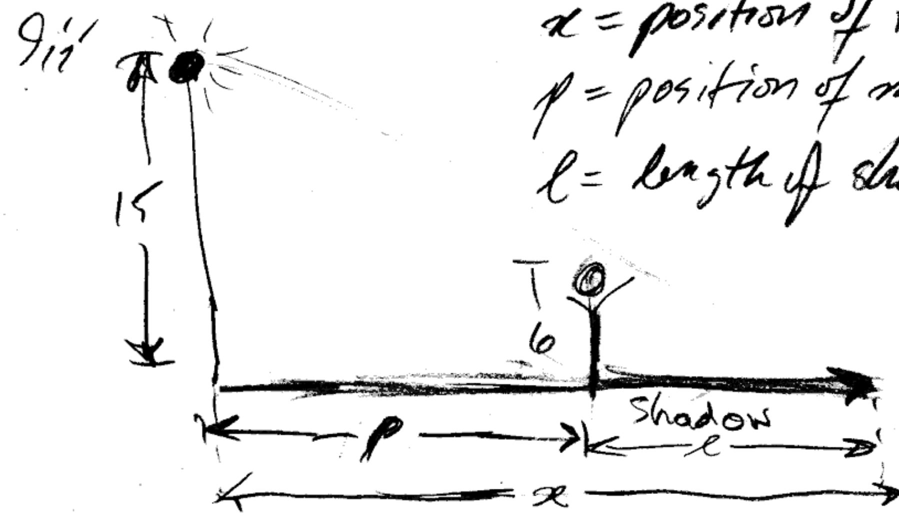
$A = xy$

$\frac{dA}{dt} = \frac{dx}{dt} \cdot y + x \frac{dy}{dt}$

$x = 15 \qquad \frac{dx}{dt} = 7 \qquad y = 6 \qquad \frac{dy}{dt} = 6$

Therefore $\frac{dA}{dt} = 7 \cdot 6 + 15 \cdot 6 = 6(22) = 132 \text{ cm}^2/\text{sec}$

x = position of tip of shadow
p = position of man
l = length of shadow



Thus $p + l = x$

Similar triangles $\frac{x}{15} = \frac{l}{6}$,

$l = \frac{6}{15}x$ $\qquad p + \frac{6}{15}x = x$

$$p = \frac{9}{15}x$$

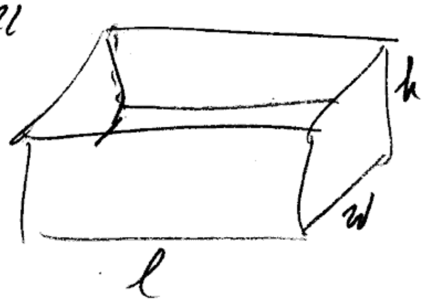
$$x = \frac{15}{9}p$$

$$\frac{dx}{dt} = \frac{15}{9}\frac{dp}{dt}$$

since $\frac{dp}{dt} = 4$ ft/sec then $\frac{dx}{dt} = \frac{15}{9} \cdot 4$ ft/sec

then $\frac{dx}{dt} = \frac{15}{9} \cdot 4 = \frac{5}{3} \cdot 4 = \frac{20}{3}$ ft/sec

9 iii



$l = 2w$

$V = lhw = 2hw^2 = 10$

$C = 10lw + 6(2wh + 2lh)$

$= 20w^2 + 6(2wh + 4wh)$

$= 20w^2 + 36wh$

From the equation for volume $h = \frac{10}{2w^2} = \frac{5}{w^2}$, therefore

$$C = 20w^2 + 36w \cdot \frac{5}{w^2}$$

$$= 20w^2 + 36 \cdot 5 \cdot w^{-1}$$

Differentiate and set to zero

$$\frac{dC}{dw} = 40w - 36 \cdot 5\, w^{-2} = 0$$

$$40w^3 = 36 \cdot 5$$

$$w^3 = \frac{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot 3 \cdot \cancel{5}}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{5}} = \frac{9}{2}$$

$$\begin{array}{r} \overset{3}{3}6 \\ \times\ 5 \\ \hline 180 \end{array}$$

$$w = \sqrt[3]{\frac{9}{2}}$$

Therefore

$$C = 20\left(\frac{9}{2}\right)^{2/3} + 36 \cdot 5 \cdot \left(\frac{2}{9}\right)^{1/3} = 20\left(\frac{9}{2}\right)^{2/3} + 180\left(\frac{2}{9}\right)^{1/3}$$

$$= 20\left(\frac{9}{2}\right)^{2/3} + 180 \cdot \frac{2}{9}\left(\frac{9}{2}\right)^{2/3} = 60\left(9/2\right)^{2/3}$$

$$= 30\left(2 \cdot 9^2\right)^{1/3} = 30\sqrt[3]{162} \approx 163.54$$