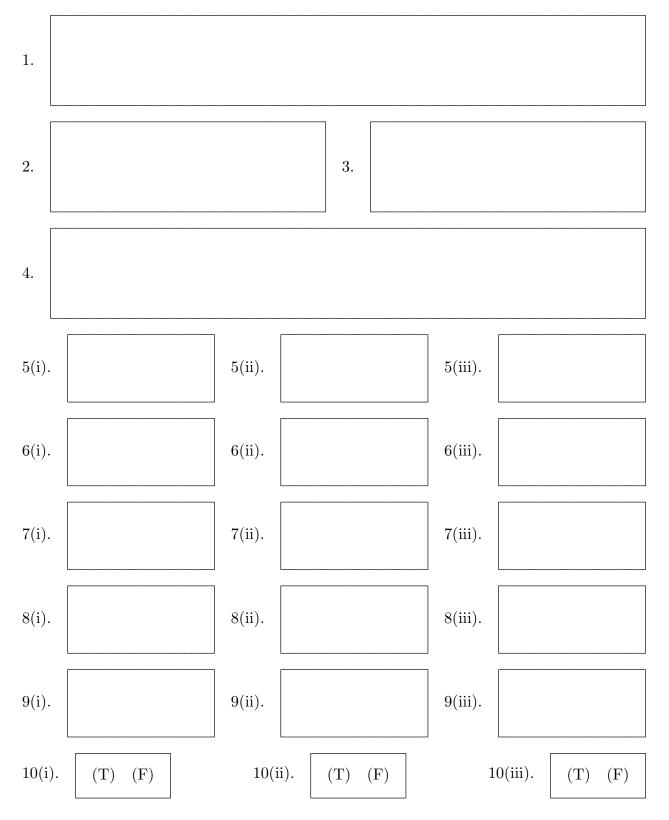Math 181 Final Review Version A

Name: _____ **Recitation:** _____

This answer sheet is the only page you will turn in. Please remove it from the rest of the test and record your answers in the spaces provided.

1.

2.   3.

4.

5(i).   5(ii).   5(iii).

6(i).   6(ii).   6(iii).

7(i).   7(ii).   7(iii).

8(i).   8(ii).   8(iii).

9(i).   9(ii).   9(iii).

10(i). (T)   (F)   10(ii). (T)   (F)   10(iii). (T)   (F)

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Math 181 Final Review Version A

1. Precisely define $\lim\limits_{x \to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Define the derivative $f'(x)$ of a function $f(x)$ using limits.

3. Suppose $x \sin y + y \sin x = 3$. Find dy/dx by implicit differentiation.

4. Define the integral $\int_a^b f(x)dx$ of a function $f(x)$ using limits.

5. Find the following limits:

   (i) $\lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x - 2}$

   (ii) $\lim\limits_{x \to \infty} \dfrac{x^2 + x - 3}{2x^2 - 4}$

   (iii) $\lim\limits_{t \to 0} \dfrac{1 - e^{-t}}{t}$

6. Find the following derivatives:

   (i) $\dfrac{d}{dx} \arctan(2x)$

   (ii) $\dfrac{d}{dx}\left(\dfrac{x}{x^2 + 7}\right)$

   (iii) $\dfrac{d}{dx}|x|^{3x}$

7. Find the following antiderivatives:

   (i) $\displaystyle\int (5x^3 - 2x^2)\, dx$

   (ii) $\displaystyle\int x^2 \cos(x^3 + 1)\, dx$

   (iii) $\displaystyle\int x\sqrt{x + 1}\, dx$

8. Compute the following areas:

   (i) $\displaystyle\int_1^2 x^2 dx$

   (ii) $\displaystyle\int_0^{\pi/2} \sin 2x\, dx$

   (iii) $\displaystyle\int_1^3 \dfrac{1}{x + 1}\, dx$

9. Solve the following story problems:

(i) The length of a rectangle is increasing at a rate of 7 cm/s and its width is increasing at a rate of 6 cm/s. When the length is 15 cm and the width is 6 cm, how fast is the area of the rectangle increasing?

(ii) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 4 ft/s along a straight path. How fast is the tip of his shadow moving when he is 35 ft from the pole?

(iii) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

10. Answer the following true/false questions:

(i) If f is differentiable at a, then f is continuous at a.

(ii) If f is continuous on $[a, b]$, then the integral $\int_a^b f(x)dx$ exists.

(iii) $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1$.