Math 181 Midterm Version A

**1.** Precisely define $\lim\limits_{x \to a^+} f(x) = L$ using inequalities in terms of $\delta$ and $\epsilon$.

**2.** Find the following limits:

**(i)** $\lim\limits_{x \to 0} \cos x$

**(ii)** $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

**(iii)** $\lim\limits_{x \to \infty} \dfrac{x^2 + x - 3}{2x^2 - 4}$

**3.** Define the derivative $f'(x)$ of a function $f(x)$ using limits.

**4.** Use the limit definition to explain why the derivative of $f(x) = 1/x$ is $f'(x) = -1/x^2$.

**5.** Answer the following true/false questions:

**(i)** If $f$ is differentiable at $a$, then $f$ is continuous at $a$.

    (A)  True

    (B)  False

**(ii)** $e$ is the number such that $\lim\limits_{h \to 0} \dfrac{e^h + 1}{h} = 1$.

    (A)  True

    (B)  False

**6.** State the following derivative rules from memory:

$$\frac{d}{dx}\sin x =$$ 

$$\frac{d}{dx}(fg)(x) =$$ 

$$\frac{d}{dx}\cos x =$$ 

$$\frac{d}{dx}(f \circ g)(x) =$$ 

$$\frac{d}{dx}\tan x =$$ 

$$\frac{d}{dx}\left(\frac{f}{g}\right)(x) =$$ 

$$\frac{d}{dx}x^\alpha =$$ 

$$\frac{d}{dx}\ln x =$$ 

$$\frac{d}{dx}\log_b x =$$ 

$$\frac{d}{dx}\arccos x =$$ 

$$\frac{d}{dx}\sec x =$$ 

$$\frac{d}{dx}e^x =$$ 

$$\frac{d}{dx}\arcsin x =$$ 

$$\frac{d}{dx}\csc x =$$ 

$$\frac{d}{dx}a^x =$$ 

$$\frac{d}{dx}\arctan x =$$ 

$$\frac{d}{dx}\cot x =$$ 

$$\frac{d}{dx}|x| =$$

**7.** Use the rules of calculus to compute the following derivatives:

(i) $\dfrac{d}{dx}(x \sin x)$

(ii) $\dfrac{d}{dx} \arctan(1 + x^2)$

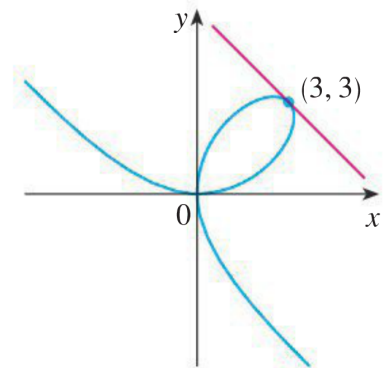(iii) $\dfrac{d}{dx}\left(\dfrac{x^3 - 5}{x^2 + 4}\right)$

(iv) $\dfrac{d}{dx} x^x$

**8.** Consider the curve defined by the equation $x^3 + y^3 = 6xy$.

   **(i)** Use implicit differentiation to find $y'$ in terms of $x$ and $y$.

   **(ii)** Find equation of the line tangent to this curve at the point $(3, 3)$.



   **(iii)** At what point in the first quadrant is the tangent line horizontal?

**9.** Two carts, A and B, are connected by a rope $39\,\text{ft}$ long that passes over a pully $P$. The point $Q$ is on the floor $12\,\text{ft}$ directly beneath $P$ and between the carts. Cart A is being pulled away from $Q$ at a speed of $2\,\text{ft/s}$. How fast is cart B moving toward $Q$ at the instant when cart A is $5\,\text{ft}$ from $Q$?