Math 182 Honors Exam 1 Version A

1. Prove the following theorem:

WEIGHTED MEAN-VALUE THEOREM FOR INTEGRALS. Assume f and g are continuous on $[a, b]$. If g never changes sign in $[a, b]$ then, for some c in $[a, b]$, we have

$$\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx.$$

2. A horse breeder plans to set aside a rectangular region of 1 square kilometer for horses and wishes to build a wooden fence to enclose the region. Since one side of the region will run along a well-traveled highway, the breeder decides to make that side more attractive, using wood that costs three times as much per meter as the wood for the other sides. What dimensions will minimize the cost of the fence?

3. $\frac{d}{dx}\left(\frac{x}{1+\sin^2 x}\right)$

4. $\frac{d}{dx}\left(\ln(x^2+1)\right)^{2x}$

5. $\frac{d}{dx}\left(\arcsin\frac{x}{2}\right)$

6. $\int_1^2 \left(x^2 - 2x + \frac{1}{x}\right) dx$

7. $\int_0^1 \arctan x \, dx$

8. $\displaystyle\int \frac{7}{x^2 - 4x + 5}\,dx$

9. $\displaystyle\int \frac{x^2}{x^2 + x - 6}\,dx$

10. [Extra Credit] A periodic function with period a satisfies $f(x+a) = f(x)$ for all x in its domain. What can you conclude about a function which has a derivative everywhere and satisfies an equation of the form

$$f(x+a) = bf(x)$$

for all x, where a and b are positive constants.