1. Fill in the missing blanks in the following theorems:

FIRST FUNDAMENTAL THEOREM OF CALCULUS. Let f be a function that is integrable on $[a, x]$ for each x in $[a, b]$. Let c be such that $a \leq c \leq b$ and define a new function A as follows

$$\boxed{\quad\quad\quad\quad\quad\quad} \quad \text{if} \quad a \leq x \leq b.$$

Then the derivative $A'(x)$ exists at each point x in the open interval (a, b) where f is continuous, and for such x we have

$$\boxed{\quad\quad\quad\quad\quad\quad}.$$

SECOND FUNDAMENTAL THEOREM OF CALCULUS. Assume f is continuous on an open interval I, and let P be any primative of $f$on I. Then, for each c and each x in I, we have

$$P(x) = P(c) + \boxed{\quad\quad\quad\quad\quad\quad}.$$

TAYLOR'S THEOREM. Assume f has a continuous derivative of order $n + 1$ in some interval containing a. Then, for every x in this interval, we have the Taylor formula

$$f(x) = \boxed{\quad\quad\quad\quad\quad\quad\quad\quad} + E_n(x),$$

where

$$E_n(x) = \frac{1}{n!} \int_a^x (x - t)^n f^{(n+1)}(t)\, dt.$$

OTHER FORM OF THE REMAINER. Using the weighted mean-value theorem for integrals $E_n(x)$ can be written as

$$E_n(x) = \boxed{\quad\quad\quad\quad\quad\quad}$$

where c lies in the closed interval joining a and x.

Newton's law of cooling states that the rate a body changes temperature is proportional to the difference between its temperature and that of the surrounding medium. If y is the temperature of the body at time t and if M denotes the temperature of the surrounding medium, Newton's law leads to the linear ordinary differential equation

$$y' + ky = kM$$

where k is a positive constant.

2. A thermometer has been stored in a room whose temperature is 80 degrees. Three minutes after being taken outdoors it reads 44 degrees. Nine minutes after being taken outdoors it reads 28 degrees. Compute the outdoor temperature.

3. Find the following antiderivatives:

(i) $\displaystyle\int \frac{1}{\sin x}\, dx$

(ii) $\displaystyle\int \frac{1}{4x^2+1}\, dx$

(iii) $\displaystyle\int \frac{1}{4x^2-1}\, dx$

(iv) $\displaystyle\int x \ln x \, dx$

4. Find the following derivatives:

(i) $\frac{d}{dx} x^{2x}$

(ii) $\frac{d}{dx} \arcsin(x^2)$

5. Solve the initial value problem $(x-1)y' = xy^2$ with initial condition $y(0) = 1$.

6. Taylor's theorem states

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + (-1)^n \frac{x^{2n+1}}{(2n+1)!}\cos c$$

for some c between 0 and x.

(i) Obtain an approximation r to the positive solution of $5x^2 = 6\sin x$ by using the cubic Taylor polynomial approximation to $\sin x$.

(ii) Use the remainder term in Taylor's formula to find a bound on $|5r^2 - 6\sin r|$ where r is the approximation found in part (i).

7. Find the following limits:

(i) $\displaystyle\lim_{x\to 0} \frac{\log(1+x)}{e^{2x}-1}$

(ii) $\displaystyle\lim_{x\to 0} \frac{1-\cos(3x)}{\sin(x^2)}$

8. Prove that if $a > 0$ and $b > 0$ then $\displaystyle\lim_{x\to\infty+} \frac{(\log x)^b}{x^a} = 0.$

9. [Extra Credit] Find the following limit:

$$\lim_{x \to 0} \frac{3\tan 4x - 12\tan x}{3\sin 4x - 12\sin x}$$