Math 182 Honors Exam 1 Version A

1. Find the following indefinite integrals:

(i) $\displaystyle\int \frac{1}{5+x^2}\,dx$

(ii) $\displaystyle\int x\sin(2x)\,dx$

(iii) $\displaystyle\int \frac{1}{1+e^x}\,dx$

2. Find the following definite integrals:

(i) $\displaystyle\int_0^2 (3+4x)\,dx$

(ii) $\displaystyle\int_0^3 \frac{1}{1+2x}\,dx$

(iii) $\displaystyle\int_0^{1/2} \arcsin x\,dx$

3. State Taylor's Theorem with the integral form of the remainder term.

4. State Taylor's forumula with remainder term expanded about $a = 0$ for the functions

(i) $\sin x$

(ii) $\arctan x$

5. Use Taylor's series to compute

(i) $\lim_{x\to 0} \frac{\sin x - x\cos x}{x^3}$.

(ii) $\lim_{x\to 0} \frac{\ln(1-x^2) + x\arctan x}{x^4}$.

6. Use the error term in Taylor's Theorem to explain what value of m will ensure that

$$\sum_{k=0}^{m} \frac{(-1)^k \left(\frac{1}{10}\right)^{2k+1}}{(2k+1)!}$$

approximates $\sin \frac{1}{10}$ to within 10^{-12}. To minimize needless computation you may use the table at the bottom of the page.

n	10^n	$n!$
1	10	1
2	100	2
3	1,000	6
4	10,000	24
5	100,000	120
6	1,000,000	720
7	10,000,000	5,040
8	100,000,000	40,320
9	1,000,000,000	362,880
10	10,000,000,000	3,628,800
11	100,000,000,000	39,916,800
12	1,000,000,000,000	479,001,600

7. Use the integral test to determine whether the following infinite series converge.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1006}$

(iii) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$