Math 182 Honors Quiz 6 Version A

1. Solve the following integration problems:

(i) $\int_0^1 xe^{-x^2}\,dx \quad = -\frac{1}{2}\int_0^{-1} e^u\,du = -\frac{1}{2}e^u\Big|_0^{-1}$

$u = -x^2$

$du = -2x\,dx \qquad = -\frac{1}{2}e^{-1} + \frac{1}{2} = \frac{1}{2}\left(1 - \frac{1}{e}\right)$

$x\,dx = -\frac{du}{2}$

(ii) $\int_2^\infty \frac{1}{x^3}\,dx = \int_2^\infty x^{-3}\,dx = \frac{1}{-2}x^{-2}\Big|_2^\infty$

$= 0 + \frac{1}{2}2^{-2} = \frac{1}{8}$

(iii) $\int \frac{1}{\sqrt{1+e^x}}\,dx$

$u = \sqrt{1+e^x} = (1+e^x)^{1/2}\,; \qquad u^2 = 1+e^x\,; \quad e^x = u^2 - 1$

$du = \frac{1}{2\sqrt{1+e^x}}\,e^x\,dx$

$\frac{2\,du}{u^2-1} = \frac{1}{\sqrt{1+e^x}}\,dx$

$\int \frac{1}{\sqrt{1+e^x}}\,dx = \int \frac{2}{u^2-1}\,du = \int\left(\frac{1}{u-1} - \frac{1}{u+1}\right)du$

$= \ln|u-1| - \ln|u+1|$

$= \ln|\sqrt{1+e^x} - 1| - \ln|\sqrt{1+e^x} + 1|$

Math 182 Honors Quiz 5 Version A

2. State 4 terms of the Taylor's series for $f(x) = (x+1)\sin x$ expanded about $a = 0$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$x \sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \dots$$

$$(x+1)\sin x = \underbrace{x + x^2 - \frac{x^3}{3!} - \frac{x^4}{3!}}_{\text{first 4 terms}} + \frac{x^5}{5!} + \frac{x^6}{5!}$$

3. Use Taylor's series to compute $\lim\limits_{x\to 0} \dfrac{2xe^x + \ln(1-2x)}{x^3}$.

$$2xe^x = 2x\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right) = \cancel{2x} + \cancel{2x^2} + x^3 + \dots$$

$$\ln(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \dots$$

$$\ln(1-2x) = \cancel{-2x} - \cancel{\frac{(2x)^2}{2}} - \frac{(2x)^3}{3} - \dots$$

$$2xe^x + \ln(1-2x) = x^3 - \frac{(2x)^3}{3} + \dots$$

$$= x^3 - \frac{8}{3}x^3 + \dots = \left(1 - \frac{8}{3}\right)x^3 + \dots$$

$$= -\frac{5}{3}x^3 + \dots$$

$$\lim_{x\to 0} \frac{2xe^x + \ln(1-2x)}{x^3} = \lim_{x\to 0} \frac{-\frac{5}{3}\cancel{x^3} + \dots}{\cancel{x^3}} = -\frac{5}{3}.$$