Math 182 Final Exam Version A

1. Convert the repeating decimal $2.\overline{63}$ to a fraction.

2. Find the following derivatives:

(i) $\dfrac{d}{dx}e^{\cosh x}$

(ii) $\dfrac{d}{dx}\ln\big(1+|x|\big)$

(iii) $\dfrac{d}{dx}(\arcsin x)^{2x}$

3. Solve the following indefinite integrals:

(i) $\displaystyle\int 2x\cos^3(1+x^2)\,dx$

(ii) $\displaystyle\int \frac{1}{x^2-5x+13}\,dx$

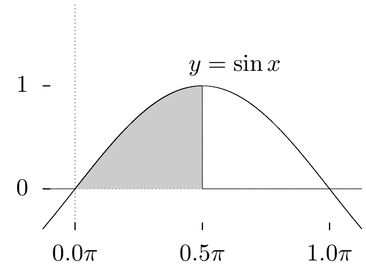
(iii) $\displaystyle\int \frac{x}{(2x-1)^2}\,dx$

4. Rewrite the integral $\int_1^2 \sqrt{x^2-1}\,dx$ in terms of u where $u = x^2$.

5. Find $\int_0^{\pi/6} x\sin(2x)\,dx$

6. Find $\int_0^{\infty} e^{-|1-x|}\,dx$

7. Find the volume generated by revolving the shaded region about the x-axis.



8. Find the length of the curve given by $y = (x/2)^{2/3}$ between $x = 0$ and $x = 2$.

9. Consider the following theorem from your book:

> Theorem 12. Let $\sum a_n$ be a series with positive terms and suppose that
> $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho.$$
> Then (a) the series converges if $\rho < 1$, (b) the series diverges if $\rho > 1$ or ρ is infinite and (c) the test is inconclusive if $\rho = 1$.

(i) What is the name of this theorem?

(ii) Establish part (a) of this theorem for the case where $\rho < 1$.

10. Determine whether the following series converge or diverge and explain your answer.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\arctan n}$

(ii) $\displaystyle\sum_{n=6}^{\infty} \frac{1}{n(\ln n)^2}$

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n!}$

11. State the Taylor series expanded about $a = 0$ along with the radius of convergence of the series for the following functions.

(i) $\dfrac{1}{1-x}$

(ii) $\sin x$

(iii) $\arctan x$

12. Find $\displaystyle\lim_{x\to 0} \frac{xe^{-x^2} - \sin x}{x^3}$