Math 182 Quiz 1 Version B

~ Key ~

1. $\frac{d}{dx} \ln \sqrt{x}$ $= \frac{d}{dx} \ln x^{1/2} = \frac{d}{dx}\left(\frac{1}{2} \ln x\right) = \frac{1}{2x}$

2. $\frac{d}{dx} |x|^5$ $= 5|x|^4 \frac{x}{|x|} = 5x|x|^3$

3. $\frac{d}{dx} \frac{\sin x}{1+x^2}$ $= \frac{(\cos x)(1+x^2) - (\sin x)(2x)}{(1+x^2)^2}$

4. $\frac{d}{dx} 3^{\sin x}$ $= (3^{\sin x})(\ln 3) \cos x$

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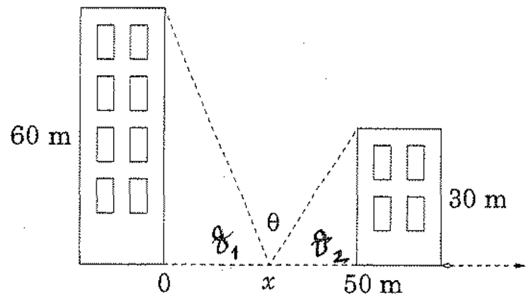
5. You are under contract to build a solar station at ground level on the east–west line between two buildings. One building is 60 meters tall, the other building is 30 meters tall and the distance between the buildings is 50 meters.



Find the value of x that maximizes θ in the diagram above to find out how far from the taller building the station should be placed to maximize the number of hours it will be in the sun on a day when the sun passes directly overhead.

$$\theta + \theta_1 + \theta_1 = \pi$$

$$\tan\theta_1 = \frac{60}{x} \qquad \tan\theta_2 = \frac{30}{50-x}$$

Put $x = 10y$ so that

$$\tan\theta_1 = \frac{6}{y} \qquad \tan\theta_2 = \frac{3}{5-y}$$

Now

$$\theta = \pi - \arctan\frac{6}{y} - \arctan\frac{3}{5-y}$$

$$\frac{d\theta}{dy} = -\frac{1}{1+\left(\frac{6}{y}\right)^2}\left(\frac{-6}{y^2}\right) - \frac{1}{1+\left(\frac{3}{5-y}\right)^2}\left(\frac{-3}{(5-y)^2}\right)(-1)$$

$$= \frac{6}{y^2+6^2} - \frac{3}{(5-y)^2+3^2} = 0$$

Thus

$$6\left((5-y)^2+3^2\right) = 3\left(y^2+6^2\right)$$

On back →

Again...

$$2\left((5-y)^2+3^2\right) = y^2+6^2$$

$$2\left(25-10y+y^2+9\right) = y^2+36$$

$$y^2-20y+50+18-36=0$$

$$\begin{array}{r} 50 \\ +18 \\ \hline 68 \\ -36 \\ \hline 32 \end{array}$$

$$y^2-20y+32=0$$

$$y = \frac{20 \pm \sqrt{(20)^2-4\cdot 32}}{2}$$

$$\begin{array}{r} 100 \\ 32 \\ \hline 68 \end{array}$$

$$= 10 \pm \sqrt{10^2-32}$$

$$= 10 \pm \sqrt{68} = 10 \pm 2\sqrt{17}$$

choo

$$4\overline{)68}^{\,17}$$

Choose $\ominus$ so it's between the buildings.

Thus $x = 100 - 20\sqrt{17}$ is the location that maximizes the amount of sun.

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6. State the Fundamental Theorem of Calculus Part 1. Suppose $f:[a,b]\to\mathbb{R}$ is a continuous function. Then for $x\in(a,b)$ the function $F(x)=\int_a^x f(t)dt$ is differentiable, and

$$F'(x)=\frac{d}{dx}\int_a^x f(t)dt = f(x).$$

7. Solve the initial value problem $\dfrac{dx}{dt}=\sqrt{t+4}$ where $x(0)=0$.

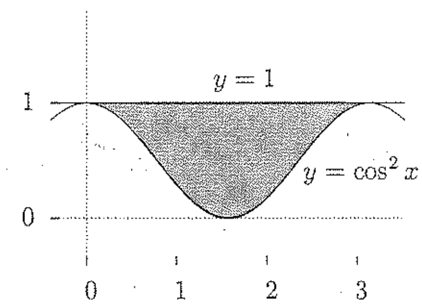
$$x=\int\sqrt{t+4}\,dt = \frac{2}{3}(t+4)^{3/2}+c$$

$$x(0)=\frac{2}{3}(4)^{3/2}+C=\frac{2}{3}2^3+C=\frac{16}{3}+C=0$$

Therefore

$$x(t)=\frac{2}{3}(t+4)^{3/2}-\frac{16}{3}$$

8. Find the total area of the shaded region given by



$$A=\int_0^\pi(1-\cos^2 x)\,dx=\int_0^\pi \sin^2 x\,dx$$

$$=\int_0^\pi\frac{1-\cos 2x}{2}\,dx=\frac{\pi}{2}-\int_0^\pi\frac{\cos 2x}{2}\,dx$$

$$=\frac{\pi}{2}-\frac{\sin 2x}{4}\Big|_0^\pi=\frac{\pi}{2}$$

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9. $\int xe^{x^2}\,dx$ $= \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$

$u = x^2$

$du = 2x\,dx$ $= \frac{1}{2}e^{x^2} + C$

$\frac{du}{2} = x\,dx$

10. $\int \frac{1}{x(\ln x)^2}\,dx$ $= \int \frac{1}{u^2}du = -\frac{1}{u} + C$

$u = \ln x$

$du = \frac{1}{x}dx$ $= -\frac{1}{\ln x} + C$

11. $\int_0^8 \sqrt{z+1}\,dz$ $= \int_1^9 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_1^9 = \frac{2}{3}(9^{3/2}-1)$

$u = z+1$

$du = dz$ $= \frac{2}{3}(3^3-1) = \frac{2}{3}(26)$

$= \frac{52}{3}$

12. $\int_0^{\pi/4} \sin x \cos x\,dx$ $= \int_0^{1/\sqrt{2}} u\,du = \frac{1}{2}u^2\Big|_0^{1/\sqrt{2}} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$u = \sin x$

$du = \cos x\,dx$