Math 182 Quiz 4 Version A

1. Find the following indefinite integrals:

(i) $\displaystyle\int \frac{x^2}{\sqrt{x+1}}\,dx$

(ii) $\displaystyle\int \frac{x^2}{x^2-1}\,dx$

(iii) $\displaystyle\int (x^2+4)\sin(x+2)\,dx$

2. Find the following improper integrals:

(i) $\displaystyle\int_0^1 \frac{x+1}{\sqrt{x}}\,dx$

(ii) $\displaystyle\int_0^\infty \frac{1}{x^2+3}\,dx$

(iii) $\displaystyle\int_1^\infty \frac{1}{x(x+1)(x+2)}\,dx$

3. Consider the following theorem from your book:

> Theorem 9. Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where f is a continuous, positive, decreasing function of x for all $x \geq N$ (N a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)\,dx$ both converge or both diverge.

(i) What is the name of this theorem?

(ii) Establish this theorem for the case $N = 1$.

4. Determine whether the following series converge or diverge and explain your answer.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^2}$

(ii) $\displaystyle\sum_{n=7}^{\infty} \frac{2n}{n^2+1}$

(iii) $\displaystyle\sum_{n=15}^{\infty} ne^{-n^2}$