Honors Math 182 Additional Study Problem For Exam 2

1. The Taylor's formula for 2^x when $a = 0$ is

$$2^x = \sum_{k=0}^{n} \frac{(\ln 2)^k}{k!} x^k + R_n(x) \qquad \text{where} \qquad R_n(x) = \frac{(\ln 2)^{n+1}}{(n+1)!} x^{n+1} 2^\xi$$

and ξ is some number between 0 and x. Use the inequality

$$0 < 2^\xi \le \max(1, 2^x) = \begin{cases} 2^x & \text{if } x > 0 \\ 1 & \text{if } x \le 0 \end{cases}$$

to demonstrate the following.

(i) Show that $R_n(5) \to 0$ as $n \to \infty$.

(ii) Estimate how large n has to be in order to guarantee $|R_n(5)| \le 0.5 \times 10^{-3}$.

(iii) Show that $R_4(x) = \mathcal{O}(x^5)$ as $x \to 0$.

(iv) Estimate how small $|x|$ has to be in order to guarantee $|R_4(x)| \le 0.5 \times 10^{-3}$.