Honors Math 182 Homework 5 Version A

1. Find the following limits

(i). $\lim_{x \to 3^+} \sin \pi x = \sin 3\pi = 0$

(ii) $\lim_{x \to 3^+} \frac{\sin \pi x}{x-3} = \lim_{y \to 0^+} \frac{\sin \pi (y+3)}{y}$

$y = x-3 \qquad y \to 0^+$ as $x \to 3^+$

$x = y+3$

$$= \lim_{y \to 0^+} \frac{\sin \pi y \cos 3\pi + \cos \pi y \sin 3\pi}{y}$$

$$= \lim_{y \to 0^+} \frac{-\sin \pi y}{y} = \lim_{z \to 0^+} \frac{-\sin z}{z/\pi}$$

$z = \pi y \qquad y \to 0^+$

$y = \frac{z}{\pi} \qquad z \to 0^+$

$$= -\pi \lim_{z \to 0^+} \frac{\sin z}{z} = -\pi$$

(iii) $\lim_{x \to 3^-} \cos \pi x = \cos 3\pi = -1$

(iv) $\lim_{x \to 3^-} \frac{\cos \pi x}{x-3} = \frac{\lim_{x \to 3^-} \cos \pi x}{\lim_{x \to 3^-} x-3} = \frac{-1}{0^-} = \infty$

Homework 5 Version A continued...

2. Find the following antiderivatives:

(i) $\int x\sqrt{4+x^4}\,dx = \int 2x\sqrt{1+\left(\frac{x^2}{2}\right)^2}\,dx$

$u = x^2/2 \qquad du = x\,dx$

$= 2\int\sqrt{1+u^2}\,du = 2\int\sqrt{1+\sinh^2 t}\,\cosh t\,dt = 2\int\cosh^2 t\,dt$

$u = \sinh t \qquad du = \cosh t\,dt$

$= \int(\cosh 2t + 1)\,dt = \frac{\sinh 2t}{2} + t = \sinh t\cosh t + t$

$= u\sqrt{1+u^2} + \operatorname{asinh} u = \frac{x^2}{2}\sqrt{1+\left(\frac{x^2}{2}\right)^2} + \operatorname{asinh}\left(\frac{x^2}{2}\right)$

(ii) $\int 2^{\sqrt{x}}\,dx$

$s = \sqrt{x} \qquad ds = \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2s}\,dx$

$= 2\int s\,2^s\,ds$

$u = s \qquad du = ds$

$dv = 2^s\,ds \qquad v = \frac{1}{\ln 2}2^s$

$= 2\left(\frac{s}{\ln 2}2^s - \frac{1}{\ln 2}\int 2^s\,ds\right)$

$= \frac{2}{\ln 2}\left(s2^s - \frac{1}{\ln 2}2^s\right)$

$= \frac{2}{\ln 2}2^s\left(s - \frac{1}{\ln 2}\right)$

$= \frac{2}{\ln 2}2^{\sqrt{x}}\left(\sqrt{x} - \frac{1}{\ln 2}\right)$

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2 iii) $\int \frac{2x^2+7x-1}{x^3+x^2-x-1}\,dx$

$$= \int \frac{2x^2+7x-1}{x^2(x+1)-(x+1)}\,dx$$

$$= \int \frac{2x^2+7x-1}{(x^2-1)(x+1)}\,dx$$

$$= \int \frac{2x^2+7x-1}{(x-1)(x+1)^2}\,dx$$

$$= \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right)dx$$

$$= A\ln|x-1| + B\ln|x+1| - \frac{C}{x+1}$$

$$A(x+1)^2 + B(x-1)(x+1) + C(x-1) = 2x^2+7x-1$$

Set $x=1$, then $4A = 2+7-1 = 8$, $A=2$

Set $x=-1$, then $-2C = 2-7-1 = -6$, $C=3$

Differentiate 2 times $2A+2B = 4$, $B = 2-A = 0$

$$= 2\ln|x-1| - \frac{3}{x+1}$$

Homework 5 version A continued

2(iv) $\int \frac{x^2+1}{x+4}\,dx$

$u = x+4 \qquad du = dx$

$x = u-4$

$= \int \frac{(u-4)^2+1}{u}\,du$

$= \int \frac{u^2-8u+17}{u}\,du$

$= \int \left(u-8+\frac{17}{u}\right)du$

$= \frac{1}{2}u^2 - 8u + 17\ln|u|$

$= \frac{1}{2}(x+4)^2 - 8(x+4) + 17\ln|x+4|$

Homework 5 version A continued...

3. Let $f(x) = \dfrac{1}{3-\sqrt{2x-5}}$

(i) Find the domain of f.

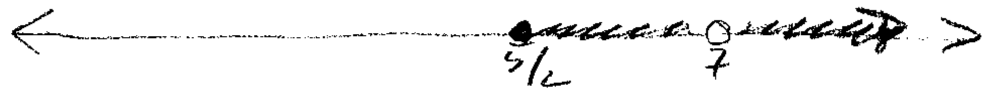
$2x-5 \geq 0$ and $3-\sqrt{2x-5} \neq 0$

$2x \geq 5$ $\qquad$ $3 \neq \sqrt{2x-5}$

$x \geq 5/2$ $\qquad$ $9 \neq 2x-5$

$2x \neq 14$

$x \neq 7$



The domain is $[5/2, 7) \cup (7, \infty)$.

(ii) Find $f'(x)$

$$f'(x) = \frac{-1}{(3-\sqrt{2x-5})^2} \cdot \frac{d}{dx}\left(3-\sqrt{2x-5}\right)$$

$$= \frac{-1}{(3-\sqrt{2x-5})^2} \cdot \left(-\frac{1}{\sqrt{2x-5}}\right)$$

$$= \frac{1}{(3-\sqrt{2x-5})^2 \sqrt{2x-5}}$$

can't be zero so don't include 5/2 for x.

for x in $(5/2, 7) \cup (7, \infty)$.

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Homework 5 version A continued...

3(iii) find the antiderivative.

$$\int f(x)\,dx = \int \frac{1}{3-\sqrt{2x-5}}\,dx =$$

$u = 2x-5 \qquad du = 2\,dx$

$$= \frac{1}{2}\int \frac{1}{3-\sqrt{u}}\,du = \frac{1}{2}\int \frac{3+\sqrt{u}}{9-u}\,du$$

$v = 9-u \qquad dv = -du$

$$= \frac{1}{2}\int \frac{3-\sqrt{9-v}}{v}\,dv$$

$$= \frac{3}{2}\int \frac{1}{v}\,dv + \frac{1}{2}\int \frac{\sqrt{9-v}}{v}\,dv$$

Homework 5 version A continued...

3(iii) Find the antiderivative.

$$\int f(x)dx = \int \frac{1}{3-\sqrt{2x-5}}\,dx$$

$$u = \sqrt{2x-5} \qquad du = \frac{1}{\sqrt{2x-5}}\,dx = \frac{1}{u}\,dx$$

$$= \int \frac{u\,du}{3-u} = \int \frac{u-3+3}{3-u}\,du$$

$$= \int \left(\frac{3}{3-u} - 1\right) du = -3\ln|3-u| - u$$

$$= -3\ln|3-\sqrt{2x-5}| - \sqrt{2x-5}$$

(iv) Find the limit

$$\lim_{b\to 7^-} \int_5^b f(x)dx = \lim_{b\to 7^-} \left(-3\ln|3-\sqrt{2x-5}| - \sqrt{2x-5}\right)\Big|_5^b$$

$$= \lim_{b\to 7^-} \left(-3\ln|3-\sqrt{2b-5}| - \sqrt{2b-5}\right) + 3\ln|3-\sqrt{5}| - \sqrt{5}$$

$$= -3\ln|3-\sqrt{9^-}| - \sqrt{9^-} + 3\ln|3-\sqrt{5}| - \sqrt{5}$$

$$= -3\ln|0^+| - 3 + 3\ln(3-\sqrt{5}) - \sqrt{5}$$

$$= -3\cdot(-\infty) - 3 + 3\ln(3-\sqrt{5}) - \sqrt{5} = \infty$$

Homework 5 version A continued...

4. Substitute $u = \arctan x$ in the following integrals.

(i) $\int_0^1 \arctan x \, dx$

$u = \arctan x \qquad x = \tan u$

$dx = \sec^2 u \, du$

$$= \int_{\arctan 0}^{\arctan 1} u \sec^2 u \, du$$

$$= \int_0^{\pi/4} u \sec^2 u \, du$$

(ii) $\int_0^{\sqrt{3}} \arctan \sqrt{x} \, dx$

$u = \arctan x \qquad x = \tan u$

$dx = \sec^2 u \, du$

$$= \int_0^{\pi/3} \arctan \sqrt{\tan u} \, \sec^2 u \, du$$

Homework 5 version A continued...

5. Define

$$S(x) = \int_0^x \sin(t^2)\,dt \quad \text{and} \quad C(x) = \int_0^x \cos(t^2)\,dt$$

Find the following derivatives.

(i) $$\frac{d}{dx}\frac{S(x^2)}{C(x)} = \frac{S'(x^2)2x\,C(x) - S(x^2)\,C'(x)}{(C(x))^2}$$

$$= \frac{2x\sin(x^4)\,C(x) - \cos(x^2)\,S(x^2)}{(C(x))^2}$$

(ii) $$\frac{d}{dx}(S\circ C)(2x) = S'(C(2x))\,C'(2x)\cdot 2$$

$$= 2\sin\left(C(2x)^2\right)\cos(4x^2)$$

$$= 2\cos(4x^2)\sin\left(C(2x)^2\right)$$

Homework 5 version A continued...

6. Let $f(t) = \cosh(t)$ and $g(t) = t$.

(i) Find the length of the curve given by $(f(t), g(t))$ where $0 \le t \le 2$.

$$L = \int_0^2 \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

$$= \int_0^2 \sqrt{\sinh^2 t + 1}\; dt = \int_0^2 \sqrt{\cosh^2 t}\; dt$$

$$= \int_0^2 \cosh t \, dt = \sinh t \Big|_0^2$$

$$= \sinh 2 - \sinh 0 = \sinh 2.$$

(ii) Find the surface area formed by revolving this curve about the y-axis.

$$A_y = \int_0^2 2\pi f(t) \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

$$= \int_0^2 2\pi \cosh(t) \cdot \cosh t \, dt$$

$$= \int_0^2 2\pi \cosh^2(t) dt = \int_0^2 2\pi \, \frac{\cosh 2t + 1}{2} \, dt$$

$$= \pi \int_0^2 (\cosh 2t + 1)\, dt = \pi \left(\frac{\sinh 2t}{2} + t \right)\Big|_0^2$$

$$= \pi \left(\frac{\sinh 4}{2} + 2 \right) \approx 49.150$$

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Homework 5 version A continued...

6(iii) Find the surface area formed by revolving this curve about the x-axis.

$$A_x = \int_0^2 2\pi g(t)\sqrt{(f'(t))^2+(g'(t))^2}\,dt$$

$$= 2\pi\int_0^2 t\cosh t\,dt$$

$u = t \qquad du = dt$

$dv = \cosh t\,dt \qquad v = \sinh t$

$$= 2\pi\left(t\sinh t\Big|_0^2 - \int_0^2 \sinh t\,dt\right)$$

$$= 2\pi\left(2\sinh 2 - \cosh t\Big|_0^2\right)$$

$$= 2\pi(2\sinh 2 - \cosh 2 + 1)$$

$$\approx 28.221$$