Honors Math 182 Homework 7 Version A

1. Find the following definite and indefinite integrals:

(i) $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\, dx$

(ii) $\displaystyle\int \tan^3 z\, dz$

(iii) $\displaystyle\int_0^1 \frac{y}{e^{2y}}\, dy$

2. For $t > 0$ define the function

$$G(t) = \int_1^t \frac{1}{x^2\sqrt{x^2+1}}\, dx.$$

(i) Solve this integral using the substitution $u = 1/x^2$.

(ii) Solve this integral using the substitution $w = \arctan x$. Simplify your answer to show it is equal to the answer found in the previous part.

(iii) Find the limits

$$\lim_{t\to\infty} G(t) \qquad \text{and} \qquad \lim_{t\to 0+} G(t).$$

3. Consider the curve $\big(f(t), g(t)\big)$ given by

$$f(t) = e^t - t, \qquad g(t) = 4e^{t/2} \qquad \text{where} \qquad -8 \le t \le 3.$$

(i) Find the length of this curve.

(ii) Find the equation of the line tangent to this curve at the point $(1, 4)$.

(iii) Find equation of the circle osculating with this curve at the point $(1, 4)$.

4. Find the volume generated by revolving the region bounded by the curves $y = 1/x$, $x = 1$, $x = 2$ and $y = 0$ about the x-axis.

5. Consider the circle given by $x^2 + 24 + y^2 = 10x$. Find the volume generated by revolving the region bounded by this circle about the y-axis.

6. Find to 5 digit accuracy the volume generated by revolving the region bounded by the curves $y = 2/(1 + 5x)$ and $y = 1 - x^2$ about the x-axis.

7. Find to 5 digit accuracy the volume generated by revolving the region bounded by the curves $y = \ln x$ and $y = x - 2$ about the y-axis.

8. Let $f(x) = \dfrac{1}{\sqrt{x}}$.

(i) Find $f'(x)$, $f''(x)$ and $f'''(x)$.

(ii) Use induction to show that the n-th derivative of f satisfies the formula

$$f^{(n)}(x) = \frac{(-1)^n}{4^n}\frac{(2n)!}{n!}x^{-\frac{2n+1}{2}}.$$