Honors Math 182 Homework 8 Version A

1. Find to 5 digit accuracy the following definite integrals:

(i) $\displaystyle\int_0^{\pi/6} x \tan x \, dx$

(ii) $\displaystyle\int_0^1 \frac{u^2 + u + 3}{u^3 - 4u^2 + 4u + 8} \, du$

(iii) $\displaystyle\int_0^{\pi/2} \sqrt{\sin y} \, dy$

(iv) $\displaystyle\int_0^{\pi} \sqrt{\tanh^2 t + \sin^2 t} \, dt$

2. The Taylor's formula for $\sinh x$ when $a = 0$ is

$$\sin x = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + R_n(x) \qquad \text{where} \qquad R_n(x) = \frac{x^{2n+3}}{(2n+3)!} \cosh \xi$$

and ξ is some number between 0 and x. Use the inequality $\cosh \xi \leq \cosh x$ to

(i) Show that $R_n(3) \to 0$ as $n \to \infty$.

(ii) Estimate how large n has to be in order to guarantee $|R_n(3)| \leq 0.5 \times 10^{-4}$.

(iii) Show that $R_2(x) = \mathcal{O}(x^7)$ as $x \to 0$.

(iv) Use the inequality $\cosh \xi \leq \cosh 3$ for $|x| \leq 3$ to estimate to 5 digit accuracy how small $|x|$ has to be in order to guarantee $|R_2(x)| \leq 0.5 \times 10^{-4}$.

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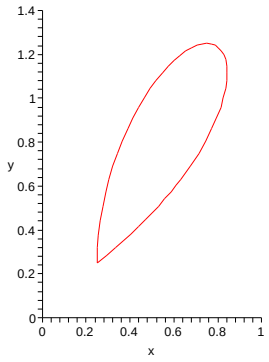
3. Consider the closed curve $(f(t), g(t))$ where $0 \leq t \leq 1$ given by

$$f(t) = \frac{1}{4} + 4t^2(1-t) \qquad \text{and} \qquad g(t) = \frac{1}{4} + \sin \pi t.$$

(i) Find to 5 digit accuracy the length of this curve.



(ii) Find to 5 digit accuracy the area enclosed by the curve.

(iii) Find the equation of the line tangent to the curve at the point $(\frac{3}{4}, \frac{5}{4})$.

(iv) Find the radius of curvature ρ of the curve at the point $(\frac{3}{4}, \frac{5}{4})$.

(v) Find the area of the surface formed by rotating this curve about the x-axis.

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4. Suppose $f(x) = \mathcal{O}(x^2)$ and $g(x) = \mathcal{O}(x^7)$ as $x \to 0$.

(i) Show $f(x) + g(x) = \mathcal{O}(x^2)$ as $x \to 0$.

(ii) Show $f(x)g(x) = \mathcal{O}(x^9)$ as $x \to 0$.

5. Find the first 3 non-zero terms of the Taylor series for e^{x^2} where $a = 0$.

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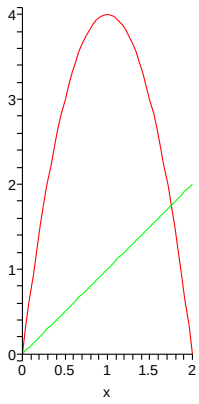
6. Consider the region enclosed by the curve $f(x) = -4x^2 + 8x$ and $g(x) = x$.

(i) Find the volume formed by rotating this region about the x-axis.



(ii) Find the volume formed by rotating this region about the y-axis.

7. Compute the following limits.

(i) $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$

(ii) $\lim_{n \to \infty} \left(n - \sqrt{n^2 + n + 3}\right)$