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> restart;
> #Q1
> I1:=Int((1+u)^(1/3),u=1..2);

$$I1 := \int_1^2 (1+u)^{1/3} du \quad (1)$$

> evalf(I1);
1.355179958 (2)
> I2:=Int((cos(x)+sin(x)^2)/(sin(x)+cos(x)^2),x=0..Pi);

$$I2 := \int_0^\pi \frac{\cos(x) + \sin(x)^2}{\sin(x) + \cos(x)^2} dx \quad (3)$$

> evalf(I2);
1.437733305 (4)
> I3:=Int(1/sqrt(1+y+y^2+y^3),y=0..1);

$$I3 := \int_0^1 \frac{1}{\sqrt{1+y+y^2+y^3}} dy \quad (5)$$

> evalf(I3);
0.7375402314 (6)
> I4:=Int(ln(t)^8,t=1..exp(1));

$$I4 := \int_1^e \ln(t)^8 dt \quad (7)$$

> evalf(I4);
0.2743615330 (8)
> #Q2
> B:=5^(2*n+2)/(2*n+2)!;

$$B := \frac{5^{2n+2}}{(2n+2)!} \quad (9)$$

> 0.5*10^(-4);
0.00005000000000 (10)
> evalf(subs(n=8,B));
0.0005958254613 (11)
> evalf(subs(n=9,B));
0.00003919904350 (12)
> #Q3
> f:=2*cos(t)+cos(5*t);
g:=3*sin(t);
df:=diff(f,t);
dg:=diff(g,t);
ddf:=diff(df,t);
ddg:=diff(dg,t);

$$f := 2 \cos(t) + \cos(5t)$$


$$g := 3 \sin(t)$$


$$df := -2 \sin(t) - 5 \sin(5t)$$


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$$\begin{aligned} dg &:= 3 \cos(t) \\ ddf &:= -2 \cos(t) - 25 \cos(5t) \\ ddg &:= -3 \sin(t) \end{aligned} \quad (13)$$

> L:=Int(sqrt(df^2+dg^2),t=0..2*Pi);

$$L := \int_0^{2\pi} \sqrt{(-2 \sin(t) - 5 \sin(5t))^2 + 9 \cos(t)^2} dt \quad (14)$$

> evalf(L);

$$25.51562749 \quad (15)$$

> A:=Int(dg*f,t=0..2*Pi);

$$A := \int_0^{2\pi} 3 \cos(t) (2 \cos(t) + \cos(5t)) dt \quad (16)$$

> evalf(A);

$$18.84955592 \quad (17)$$

> t0:=arcsin(sqrt(2)/2);

$$t0 := \frac{1}{4} \pi \quad (18)$$

> m:=subs(t=t0,dg/df);

$$m := \frac{3 \cos\left(\frac{1}{4} \pi\right)}{-2 \sin\left(\frac{1}{4} \pi\right) - 5 \sin\left(\frac{5}{4} \pi\right)} \quad (19)$$

> simplify(m);

$$1 \quad (20)$$

> kappa:=(ddg*df-dg*ddf)/(df^2+dg^2)^(3/2);

$$\kappa := \frac{-3 \sin(t) (-2 \sin(t) - 5 \sin(5t)) - 3 \cos(t) (-2 \cos(t) - 25 \cos(5t))}{((-2 \sin(t) - 5 \sin(5t))^2 + 9 \cos(t)^2)^{3/2}} \quad (21)$$

> simplify(subs(t=t0,kappa));

$$-\frac{13}{9} \quad (22)$$

> #Q6

> f:=4*sin(x);

g:=sqrt(6)-sqrt(2);

$$\begin{aligned} f &:= 4 \sin(x) \\ g &:= \sqrt{6} - \sqrt{2} \end{aligned} \quad (23)$$

> a:=solve(f=g,x);

$$a := \arcsin\left(\frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2}\right) \quad (24)$$

> evalf(a-Pi/12);

$$2.10^{-10} \quad (25)$$

> # The exact value of a
a:=Pi/12;

$$a := \frac{1}{12} \pi \quad (26)$$

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> b:=Pi-a;
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$$b := \frac{11}{12} \pi \quad (27)$$

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> Vx:=Int(Pi*(f^2-g^2),x=a..b);
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$$Vx := \int_{\frac{1}{12} \pi}^{\frac{11}{12} \pi} \pi \left(16 \sin(x)^2 - (\sqrt{6} - \sqrt{2})^2 \right) dx \quad (28)$$

```
> Vx2:=value(Vx);
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$$Vx2 := 16 \pi \sin\left(\frac{1}{12} \pi\right) \cos\left(\frac{1}{12} \pi\right) + \frac{10}{3} \sqrt{3} \pi^2 \quad (29)$$

```
> trig1:=sin(Pi/12)=sqrt((1-cos(Pi/6))/2);
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trig2:=cos(Pi/12)=sqrt((1+cos(Pi/6))/2);
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$$\text{trig1} := \sin\left(\frac{1}{12} \pi\right) = \frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2}$$

$$\text{trig2} := \cos\left(\frac{1}{12} \pi\right) = \frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2} \quad (30)$$

```
> Vx3:=subs([trig1,trig2],Vx2);
```

$$Vx3 := 16 \pi \left(\frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} \right) \left(\frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2} \right) + \frac{10}{3} \sqrt{3} \pi^2 \quad (31)$$

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> simplify(Vx3);
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$$4 \pi + \frac{10}{3} \sqrt{3} \pi^2 \quad (32)$$

```
> evalf(Vx3);
```

$$69.54855822 \quad (33)$$

```
> Vy:=Int(2*Pi*x*(f-g),x=a..b);
```

$$Vy := \int_{\frac{1}{12} \pi}^{\frac{11}{12} \pi} 2 \pi x \left(4 \sin(x) - \sqrt{6} + \sqrt{2} \right) dx \quad (34)$$

```
> Vy2:=value(Vy);
```

$$Vy2 := \frac{5}{6} \sqrt{2} \pi^3 + 8 \cos\left(\frac{1}{12} \pi\right) \pi^2 - \frac{5}{6} \sqrt{2} \sqrt{3} \pi^3 \quad (35)$$

```
> Vy3:=subs([trig1,trig2],Vy2);
```

$$Vy3 := \frac{5}{6} \sqrt{2} \pi^3 + 8 \left(\frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2} \right) \pi^2 - \frac{5}{6} \sqrt{2} \sqrt{3} \pi^3 \quad (36)$$

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> factor(Vy3);
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$$\frac{1}{6} \pi^2 \left(5 \sqrt{2} \pi - 5 \sqrt{2} \sqrt{3} \pi + 12 \sqrt{6} + 12 \sqrt{2} \right) \quad (37)$$

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> evalf(Vy3);
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$$49.51639654 \quad (38)$$

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> evalf(sqrt(2)/6*Pi^2*(5*Pi*(1-sqrt(3))+12*(1+sqrt(3))));
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$$49.51639658 \quad (39)$$

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> #Q7
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> limit((sin(x)-x*cos(x))/x^3,x=0);
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$\frac{1}{3}$

(40)