```
> restart;
> f:=t;
  g:=sqrt(9-t^2);
```

$$f := t$$

$$g := \sqrt{9 - t^2} \tag{1}$$

```
> df:=diff(f,t);
  dg:=diff(g,t);
```

$$df := 1$$

$$dg := -\frac{t}{\sqrt{9 - t^2}} \tag{2}$$

```
> ddf:=diff(df,t);
  ddg:=diff(dg,t);
```

$$ddf := 0$$

$$ddg := -\frac{t^2}{\left(9 - t^2\right)^{3/2}} - \frac{1}{\sqrt{9 - t^2}} \tag{3}$$

```
> kappa:=(df*ddg-dg*ddf)/(df^2+dg^2)^(3/2);
```

$$\kappa := \frac{-\dfrac{t^2}{\left(9 - t^2\right)^{3/2}} - \dfrac{1}{\sqrt{9 - t^2}}}{\left(1 + \dfrac{t^2}{9 - t^2}\right)^{3/2}} \tag{4}$$

```
> simplify(kappa) assuming t<3,t>-3;
```

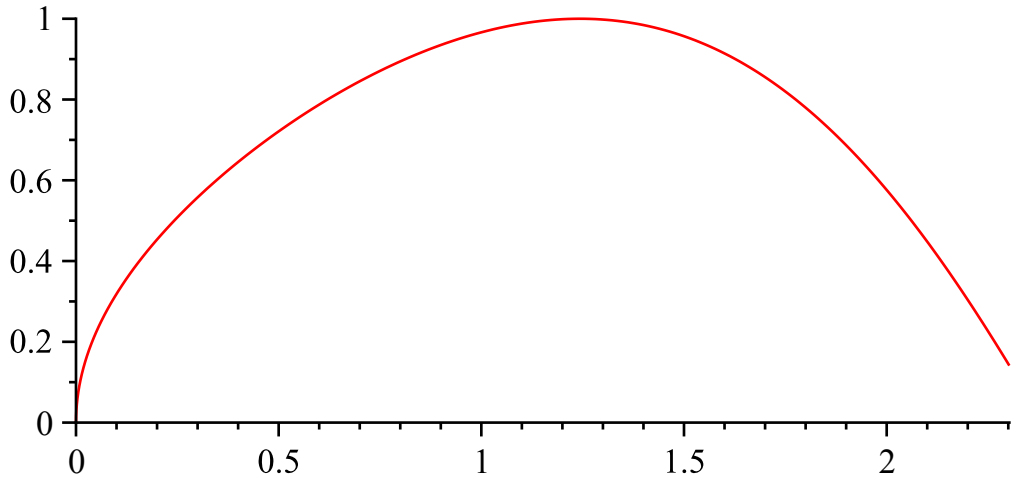
$$-\frac{1}{3} \tag{5}$$

```
> f:=ln(1+t^2);
  g:=sin(t);
```

$$f := \ln\left(1 + t^2\right)$$

$$g := \sin(t) \tag{6}$$

```
> with(plots):
> P1:=plot([f,g,t=0..3]):
> display(P1,scaling=constrained);
```




```
> f0:=subs(t=1,f);
  g0:=subs(t=1,g);
```

$$f0 := \ln(2)$$
$$g0 := \sin(1) \quad \textbf{(7)}$$

```
> df:=diff(f,t);
  dg:=diff(g,t);
  ddf:=diff(df,t);
  ddg:=diff(dg,t);
```

$$df := \frac{2\,t}{1+t^2}$$
$$dg := \cos(t)$$
$$ddf := \frac{2}{1+t^2} - \frac{4\,t^2}{(1+t^2)^2}$$
$$ddg := -\sin(t) \quad \textbf{(8)}$$

```
> kappa:=(df*ddg-dg*ddf)/(df^2+dg^2)^(3/2);
```

(9)

$$\kappa := \frac{-\frac{2\,t\sin(t)}{1+t^2} - \cos(t)\left(\frac{2}{1+t^2} - \frac{4\,t^2}{(1+t^2)^2}\right)}{\left(\frac{4\,t^2}{(1+t^2)^2} + \cos(t)^2\right)^{3/2}} \tag{9}$$

```
> kappa0:=subs(t=1,kappa);
```

$$\kappa 0 := -\frac{\sin(1)}{(1+\cos(1)^2)^{3/2}} \tag{10}$$

```
> rho0:=1/abs(kappa0);
```

$$\rho 0 := \frac{(1+\cos(1)^2)^{3/2}}{\sin(1)} \tag{11}$$

```
> den:=sqrt(df^2+dg^2);
```

$$den := \sqrt{\frac{4\,t^2}{(1+t^2)^2} + \cos(t)^2} \tag{12}$$

```
> N:=[-dg/den,df/den];
```

$$N := \left[-\frac{\cos(t)}{\sqrt{\frac{4\,t^2}{(1+t^2)^2} + \cos(t)^2}}, \frac{2\,t}{(1+t^2)\sqrt{\frac{4\,t^2}{(1+t^2)^2} + \cos(t)^2}}\right] \tag{13}$$

```
> N0:=subs(t=1,N);
```

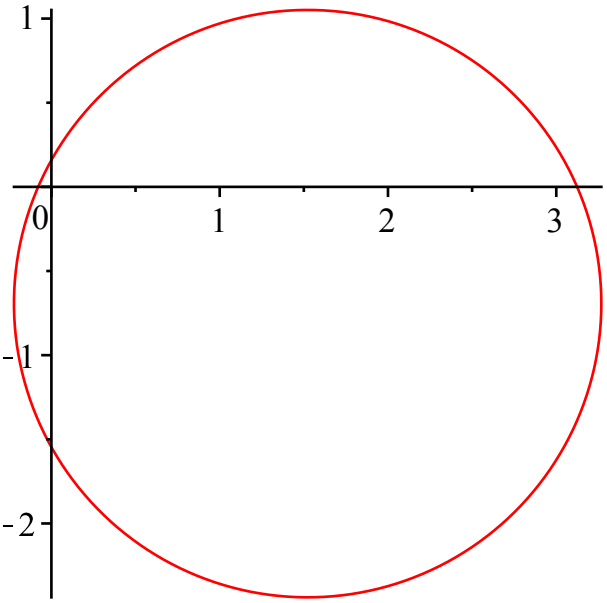
$$N0 := \left[-\frac{\cos(1)}{\sqrt{1+\cos(1)^2}}, \frac{1}{\sqrt{1+\cos(1)^2}}\right] \tag{14}$$

```
> #Since curvature was negative subtract so the circle
  #is on the right side of the curve.
  a:=f0-rho0*N0[1];
  b:=g0-rho0*N0[2];
```

$$a := \ln(2) + \frac{(1+\cos(1)^2)\cos(1)}{\sin(1)}$$

$$b := \sin(1) - \frac{1+\cos(1)^2}{\sin(1)} \tag{15}$$

```
> P2:=plot([a+rho0*cos(theta),b+rho0*sin(theta),theta=0..2*Pi]):
  display(P2,scaling=constrained);
```




```
> display([P1,P2],scaling=constrained);
```