Honors Math 182 Final Version A

1. State Taylor's formula using big-$\mathcal{O}$ for the remainder where $a = 0$ for the functions

(i) e^x

(ii) $\sin x$

(iii) $\cos x$

(iv) $\log(1 - x)$

(v) $\arctan x$

(vi) $(1 + x)^\alpha$

2. Find the following derivatives:

(i) $\dfrac{d}{dx}|\sinh 2x|^2$

(ii) $\dfrac{d}{dx}\dfrac{x}{1+e^x}$

(iii) $\dfrac{d}{dx}\arctan(1+x^2)$

(iv) $\dfrac{d}{dx}(2+\sin x)^{\ln x}$

3. Solve the following indefinite integrals:

(i) $\displaystyle\int \ln(3x)\,dx$

(ii) $\displaystyle\int \frac{x^2+17}{x+3}\,dx$

(iii) $\displaystyle\int x\sin^2 x\,dx$

(iv) $\displaystyle\int \frac{x}{\sqrt[3]{x+1}}\,dx$

4. Solve the following definite integrals:

(i) $\displaystyle\int_0^1 \frac{1}{(x+1)(x-2)}\,dx$

(ii) $\displaystyle\int_{-4}^1 x\sqrt{x+8}\,dx$

(iii) $\displaystyle\int_0^1 \frac{1}{1+e^x}\,dx$

(iv) $\displaystyle\int_0^{\pi} |\sin 2x| \cos x\,dx$

5. Use Taylor's series or L'Hôpital's rule to find the following limits if they exist.

(i) $\lim_{x \to 0} \frac{2xe^x + \ln(1-2x)}{x^3}$

(ii) $\lim_{x \to 0} \frac{\ln(1+x^2) - x^2 \cos}{x^6}$

(iii) $\lim_{x \to 0} \frac{\ln(1-x^2) + x \arctan x}{x^4}$

(iv) $\lim_{x \to 0} \frac{\ln(1+x^2) + x^2 \cos}{x^6}$

6. Show that $\sum_{n=1}^{\infty} \frac{1}{n+13} = \infty$ by comparing the sum with a suitable integral.

7. Show that $\sum_{n=1}^{\infty} \frac{n^2}{2^n} < \infty$ by using the ratio test.

8. Give an example of a series of the form $\sum_{n=1}^{\infty} (-1)^n a_n$ where $a_n > 0$ that converges.

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10. Find the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

11. Find the volume generated by revolving the region bounded by the curves $y = x^2$ and $y^2 = 8x$ around the x axis.

12. Find the curvature κ and radius of curvature ρ at the point $(e, 1)$ on the curve given by $\big(f(t), g(t)\big)$ where $f(t) = e^t$ and $g(t) = \sin(\pi t/2)$ where $0 \leq t \leq 2$

13. Comparison with the geometric series yields

> **Theorem.** If there is N large enough such that $|a_n| < cq^n$ for some $q \in (0,1)$ and all $n \geq N$, then $\sum_{n=1}^{\infty} a_n$ converges.

Use this theorem to prove one of the following corollaries:

(i) Ratio Test. Suppose $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$ where $L < 1$. Then $\sum_{n=1}^{\infty} a_n$ converges.

(ii) Root Test. Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$ where $L < 1$, Then $\sum_{n=1}^{\infty} a_n$ converges.

14. Consider the curve $\big(f(t), g(t)\big)$ given by

$$f(t) = t\cos t \qquad \text{and} \qquad g(t) = \sqrt{t}\sin t \qquad \text{where} \qquad 0 \le t \le 2\pi.$$

Find the equation of the line tangent to the curve at the point $\big(f(\pi/6), g(\pi/6)\big)$.

15. Find the surface of revolution generated by revolving the arc $y = \frac{1}{4}x^4 + \frac{1}{8}x^{-2}$ where $1 \le x \le 2$ about the y-axis.

16. Find the area enclosed by the parametric curve $\big(f(t), g(t)\big)$ given by $f(t) = \sin t$ and $g(t) = \cos t$ where $0 \le t \le 2\pi$.