~ Key ~

Honors Math 182 Quiz 6 Version A

1. Solve the following antiderivative problems:

(i) $\int \ln(2x+1)\,dx = \frac{1}{2}\int \ln u\,du = \frac{1}{2}(u\ln u - u)$

$u = 2x+1 \quad du = 2\,dx$

$= \frac{1}{2}\big((2x+1)\ln(2x+1) - (2x+1)\big) + C$

(ii) $\int \frac{x+3}{x^3-2x^2+x-2}\,dx = \int \frac{x+3}{(x-2)(x^2+1)}\,dx = \int\left(\frac{A}{x-2} + \frac{Bx+D}{x^2+1}\right)dx$

$= A\ln|x-2| + \frac{B}{2}\ln(x^2+1) + D\arctan x + C$

solve for A, B and D:

$x+3 = A(x^2+1) + (Bx+D)(x-2) = (A+B)x^2 + (D-2B)x + A - 2D$

$$\begin{cases} A+B=0 & B=-A & & A=1 \\ D-2B=1 & 2A+D=1 & 4A+2D=2 & D=-1 \\ A-2D=3 & A-2D=3 & 5A=5 & B=-1 \end{cases}$$

$\boxed{\ln|x-2| - \frac{1}{2}\ln(x^2+1) - \arctan x + C}$

(iii) $\int \sin^3 x\,dx$

$= \int (1-\cos^2 x)\sin x\,dx = \int \sin x\,dx - \int \cos^2 x \sin x\,dx$

$u = \cos x \quad du = -\sin x\,dx$

$= -\cos x + \int u^2\,du = -\cos x + \frac{1}{3}u^3 + C$

$= -\cos x + \frac{1}{3}\cos^3 x + C.$

(iv) $\int \arcsin(2x)\,dx$

$u = \arcsin 2x \quad du = \frac{2}{\sqrt{1-4x^2}}\,dx$

$dv = dx \quad v = x$

$= x\arcsin 2x - \int \frac{2x}{\sqrt{1-4x^2}}\,dx = x\arcsin 2x + \frac{1}{4}\int w^{-1/2}\,dw$

$w = 1-4x^2 \quad dw = -8x\,dx$

$= x\arcsin 2x + \frac{1}{2}w^{1/2} + C = x\arcsin 2x + \frac{1}{2}(1-4x^2)^{1/2} + C.$

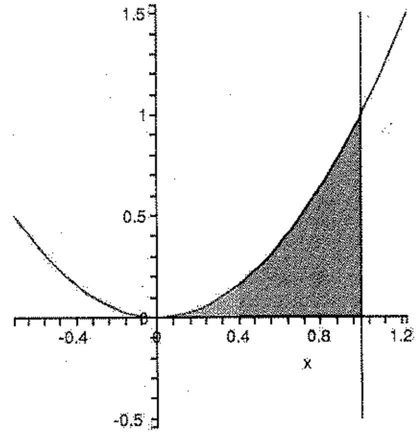
2. The area bounded by the curves $y = x^2$, $y = 0$ and $x = 1$ is depicted below.

(i) Find the volume formed by rotating this area about the x-axis.

$$V_x = \int_0^1 \pi (f(x))^2 \, dx$$

$$= \int_0^1 \pi x^4 \, dx = \pi \frac{x^5}{5} \Big|_0^1$$

$$= \frac{\pi}{5}.$$



(ii) Find the volume formed by rotating this area about the y-axis.

$$V_y = \int_0^1 2\pi x f(x) \, dx = \int_0^1 2\pi x^3 \, dx$$

$$= 2\pi \frac{x^4}{4} \Big|_0^1 = \frac{\pi}{2}.$$