Honors Math 182 Exam 2 Version B

Feel free to use the computers, your calculator, notes and textbooks while working on this exam. You may also use online resources such as Wikipedia, Google and Wolfram Alpha; however, do not use email or any other messaging service during the exam.

1. Newton's method for solving the equation $f(x) = 0$ is given by

$$x_{n+1} = \phi(x_n) \qquad \text{where} \qquad \phi(x) = x - \frac{f(x)}{f'(x)}$$

and x_0 is an initial guess to the solution.

(i) Find $\phi(x)$ in the case $f(x) = x + 4\cos x$.

(ii) Let $x_0 = 2$ and compute x_1, x_2, x_3 and x_4.

(iii) Let $x_0 = 4$ and compute x_1, x_2, x_3 and x_4.

(iv) Do the sequences in part (ii) and part (iii) converge to the same limit? If so, explain why; if not, explain why not.

2. Taylor's Theorem implies $\sin(x) = S_n(x) + R_n(x)$ where

$$S_n(x) = \sum_{l=0}^{n} (-1)^l \frac{x^{2l+1}}{(2l+1)!} \quad \text{and} \quad R_n(x) = (-1)^{n+1} \int_0^x \frac{(x-t)^{2n+2}}{(2n+2)!} \cos t \, dt.$$

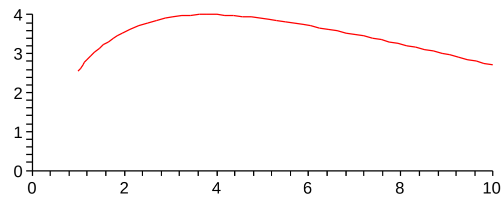
(i) Compute $S_n(2)$ for $n = 1, 2, 3, 4$.

(ii) Compute $2S_n(1)\sqrt{1 - S_n(1)^2}$ for $n = 1, 2, 3, 4$.

(iii) Prove the sequences in parts (i) and (ii) above converge to the same limit.

(iv) Which sequence converges faster? Why?

3. Consider the the curve $y = 4\sin(\ln(1+x))$ where $x \in [1, 10]$.



(i) Find the length of this curve.

(ii) Find the volume of revolution under this curve about the x-axis.

(iii) Find the volume of revolution under this curve about the y-axis.

4. Your iron works has contracted to design and build a 2000 ft^3, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding thin stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible.

(i) What dimensions do you tell the shop to use?

(ii) Briefly describe how you took weight into account.

5. Suppose that f is continuous and ϕ is continuous with continuous derivative ϕ'. Prove the change of variables formula

$$\int_{\phi(\alpha)}^{\phi(\beta)} f(t)\,dt = \int_{\alpha}^{\beta} f\big(\phi(t)\big)\phi'(t)\,dt.$$

6. Solve the following differential equation

$$\begin{cases} y' - y = x^2 \\ y(1) = 3. \end{cases}$$