Honors Math 182 Final Version A

1. State Taylor's formula with remainder where $a = 0$ and $b = x$ for the functions

(i) e^x

(ii) $\sin x$

(iii) $\cos x$

(iv) $\log(1 - x)$

(v) $\arctan x$

(vi) $(1 + x)^\alpha$

2. Find the following derivatives:

(i) $\dfrac{d}{dx}|\sinh 2x|^2$

(ii) $\dfrac{d}{dx}\dfrac{x}{1+e^x}$

(iii) $\dfrac{d}{dx}\arctan(1+x^2)$

(iv) $\dfrac{d}{dx}(2+\sin x)^{\ln x}$

3. Solve the following indefinite integrals:

(i) $\displaystyle\int \ln(3x)\,dx$

(ii) $\displaystyle\int \frac{x^2+17}{x+3}\,dx$

(iii) $\displaystyle\int x\sin^2 x\,dx$

(iv) $\displaystyle\int \frac{x}{\sqrt[3]{x+1}}\,dx$

4. Solve the following definite integrals:

(i) $\displaystyle\int_0^1 \frac{1}{(x+1)(x-2)}\,dx$

(ii) $\displaystyle\int_{-4}^1 x\sqrt{x+8}\,dx$

(iii) $\displaystyle\int_0^1 \frac{1}{1+e^x}\,dx$

(iv) $\displaystyle\int_0^\pi |\sin 2x|\cos x\,dx$

5. Use Taylor's series or L'Hôpital's rule to find the following limits if they exist.

(i) $\lim_{x \to 0} \dfrac{2xe^x + \ln(1-2x)}{x^3}$

(ii) $\lim_{x \to 0} \dfrac{\ln(1+x^2) - x^2 \cos x}{x^6}$

(iii) $\lim_{x \to 0} \dfrac{\ln(1-x^2) + x \arctan x}{x^4}$

(iv) $\lim_{x \to 0} \dfrac{\ln(1-x^2) + x^2 \cos x}{x^6}$

6. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+13} = \infty$ by comparing the sum with a suitable integral.

7. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n} < \infty$ by comparing the sum with a suitable integral.

8. Give an example of a series of the form $\displaystyle\sum_{n=1}^{\infty} (-1)^n a_n$ where $a_n > 0$ that converges.

9. Give an example of a series of the form $\displaystyle\sum_{n=1}^{\infty} (-1)^n a_n$ where $a_n > 0$ that diverges.

10. Find the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

11. Find the volume generated by revolving the region bounded by the curves $y = x^2$ and $y^2 = 8x$ around the x axis.

12. Find the surface of revolution generated by revolving the arc $y = \frac{1}{4}x^4 + \frac{1}{8}x^{-2}$ where $1 \leq x \leq 2$ about the y-axis.

13. Prove one of the following theorems:

Theorem 8. Let a_k be a sequence such that $a_k \geq 0$ for $k = 1, 2, \ldots$ and assume there is a number $s > 0$ such that $a_k^{1/k} \leq s$ for all but a finite number of integers k. Let $r < 1/s$ and $r > 0$. Then the series $\sum_{k=1}^{\infty} a_k r^k$ converges.

Theorem 9. Let a_k be a sequence such that $a_k \geq 0$ for $k = 1, 2, \ldots$ and assume there is a number $s > 0$ such that $a_k^{1/k} \geq s$ for infinitely many integers k. Let $r > 1/s$. Then the series $\sum_{k=1}^{\infty} a_k r^k$ diverges.

14. A cone-shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.