Honors Math 182 Homework 6 Version A

**1.** Find the solution to each of the following differential equations:

**(i)** $\begin{cases} y' + 2y = e^{-x} \\ y(0) = 3 \end{cases}$

**(ii)** $\begin{cases} y' - y = \sin^2(x) \\ y(0) = -1 \end{cases}$

**(iii)** $\begin{cases} y' + y = \dfrac{1}{1+x^2} \\ y(0) = 0 \end{cases}$

**(iv)** $\begin{cases} y' - 2xy = x \\ y(0) = 1 \end{cases}$

**2.** Find the volumes of revolution of the following:

**(i)** Under $y = \sec x$ between $x = 0$ and $x = \pi/4$ rotated around the $x$-axis.

**(ii)** Under $y = \sin x$ between $x = 0$ and $x = \pi/4$ rotated around the $x$-axis.

**(iii)** Under $y = \sqrt{1+x}$ between $x = 1$ and $x = 5$ rotated around the $x$-axis.

**(iv)** Under $y = \sec x$ between $x = 0$ and $x = \pi/4$ rotated around the $y$-axis.

**(v)** Under $y = \sin x$ between $x = 0$ and $x = \pi/4$ rotated around the $y$-axis.

**(vi)** Under $y = \sqrt{1+x}$ between $x = 1$ and $x = 5$ rotated around the $y$-axis.

**3.** Use Taylor's formula

$$\ln(1-x) = -\sum_{k=1}^{n} \frac{x^k}{k} - \int_0^x \frac{t^{n+1}}{1-t}\,dt$$

to approximate $\ln(4)$ in the following ways:

**(i)** Set $x = 3/4$ and compute

$$S_n = \sum_{k=1}^{n} \frac{(3/4)^k}{k}$$

for values of $n = 1, 2, \ldots, 10$.

**(ii)** Set $x = 1/2$ and compute

$$T_n = 2\sum_{k=1}^{n} \frac{(1/2)^k}{k}$$

for values of $n = 1, 2, \ldots, 10$.

**(iii)** Which method works better? Explain why.