Honors Math 182 Quiz 7 Version A

Feel free to use the computers, your calculator, notes and textbooks while working on this quiz. You may also use online resources such as Wikipedia, Google and Wolfram Alpha; however, do not use email or any other messaging service during the quiz.

1. Newton's method for solving the equation $f(x) = 0$ is given by

$$x_{n+1} = \phi(x_n) \qquad \text{where} \qquad \phi(x) = x - \frac{f(x)}{f'(x)}$$

and x_0 is an initial guess to the solution.

(i) Find $\phi(x)$ in the case $f(x) = x^2 - 4$.

(ii) Let $x_0 = 1$ and compute x_1, x_2, x_3 and x_4.

(iii) The exact solutions of $f(x) = 0$ are $x = \pm 2$. Find the difference between x_4 and the exact solution which is closest to x_4.

2. Consider the integral

$$\int_1^x \frac{1}{t}\, dt.$$

    (i) Make the change of variables $u = 1/t$ in the above integral.

    (ii) Further change the variables to $v = u^2$ in the integral obtained in part (i).

3. Taylor's Theorem implies the exponential function may be calculated from the limit

$$e^x = \lim_{n\to\infty} S_n \qquad \text{where} \qquad S_n = \sum_{k=0}^{n} \frac{x^k}{k!}.$$

    (i) Approximate $\sqrt{e}$ by computing S_2, S_4 and S_8 when $x = 1/2$.