Math 285 Exam 1 Version A

1. State Euler's method for solving the ordinary differential equation $y' = f(x, y)$ with the initial condition $y(x_0) = y_0$.

2. Given n let $h = (b - x_0)/n$ and $x_i = x_0 + hi$. The fourth-order Runge-Kutta method for solving $y' = f(x, y)$ such that $y(x_0) = y_0$ is given by

$$\begin{cases} k_1 = hf(x_i, y_i) \\ k_2 = hf(x_i + h/2, y_i + k_1/2) \\ k_3 = hf(x_i + h/2, y_i + k_2/2) \\ k_4 = hf(x_i + h, y_i + k_3) \\ y_{i+1} = y_i + (1/6)(k_1 + 2k_2 + 2k_3 + k_4) \end{cases}$$

Explain what it means in terms of the absolute error $|y(b) - y_n|$ and the step-size h that this method is fourth-order.

3. Find the critical points of the autonomous first-order differential equation

$$\frac{dy}{dx} = (y - 3)(y - 2)^2(y - 1).$$

Classify each critical point as asymptotically stable, unstable or semi-stable.

4. Find an explicit solution to

$$\frac{dy}{dx} - 2y = 1 + x \qquad \text{where} \qquad y(0) = 1.$$

5. Solve the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 1}{y + 1} \qquad \text{where} \qquad y(0) = 1.$$

Find the exact value of $y(1)$.

6. Determine whether the differential equation

$$(e^{2y} - y \cos xy)dx + (2xe^{2y} - x \cos xy + 2y)dy = 0$$

is exact.

7. Solve the initial value problem

$$ty' + 2y = \sin t \qquad \text{where} \qquad y(\pi/2) = 1.$$

8. Solve the initial value problem

$$y' = xy^3(1+x^2)^{1/2} \qquad \text{where} \qquad y(0) = 1.$$

9. Find the critical points of the autonomous first-order differential equation

$$\frac{dy}{dx} = y^3 - 9y$$

Classify each critical point as asymptotically stable, unstable or semi-stable.

10. Solve the equation

$$y\,dx + (2x - ye^y)\,dy = 0$$

Hint: Make it exact by using an integrating factor $\mu = \mu(y)$.

11. Find the general solution to the second order differential equation $y'' - 2y' + y = 0$.

12. Solve the second order initial value problem

$$y'' - 6y' + 10y = 0 \qquad \text{where} \qquad y(0) = 1 \qquad \text{and} \qquad y'(0) = 3.$$

13. Find the general solution to the initial value problem $y' = \sin(x + y)$.
Hint: Try the substitution $v = x + y$.

14. Solve the homogeneous initial value problem

$$(x^2 + 2y^2)dx - xydy = 0 \qquad \text{where} \qquad y(-1) = 1.$$

15. The function $y_1 = x^4$ is one solution to the second order differential equation

$$x^2y'' - 7xy' + 16y = 0.$$

Find a second linearly independent solution y_2.
Hint: Substitute $y_2 = vy_1$ into the differential equation and then solve for v.

16. State Theorem 1.1 from the text on the existence and uniqueness of solutions to the initial value problem

$$\frac{dy}{dx} = f(x, y) \qquad \text{where} \qquad y(x_0) = y_0.$$