Math 285 Exam 2 Review Sheet

1. Solve the initial value problem

$$y'' - 2y' + y = 0, \quad y(0) = 5, \quad y'(0) = 10.$$

2. Solve the differential equation

$$y'' + 2y' - 24y = 16 - (x+2)e^{4x}$$

by the method of undetermined coefficients.

3. Solve the differential equation

$$y'' + 2y' + y = e^{-t} \ln t$$

by variation of parameters.

4. Write the system of linear differential equations

$$\begin{cases} \dfrac{dx}{dt} = x - y + z + t - 1 \\ \dfrac{dy}{dt} = 2x + y - z - 3t^3 \\ \dfrac{dz}{dt} = x + y + z + t^2 - t + 2 \end{cases}$$

in the matrix form $X' = AX + F(t)$. What is X? What is A? What is $F(t)$?

5. The matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

has eigenvectors

$$\xi_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \text{and} \qquad \xi_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

with corresponding eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$. Solve the initial value problem

$$x' = Ax, \qquad x(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

6. The matrix

$$A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$$

has eigenvectors

$$\xi_1 = \begin{bmatrix} 1 \\ 1/2 + i/2 \end{bmatrix} \qquad \text{and} \qquad \xi_2 = \begin{bmatrix} 1 \\ 1/2 - i/2 \end{bmatrix}$$

with corresponding eigenvalues $\lambda_1 = -2 + i$ and $\lambda_2 = -2 - i$. Find the solution to the initial value problem

$$X' = AX, \qquad x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

7. Solve the nonhomogeneous system

$$X' = \begin{bmatrix} -3 & 1 \\ 2 & -4 \end{bmatrix} X + \begin{bmatrix} 3t \\ e^{-t} \end{bmatrix}.$$

Hint: The solutions to the homogenous problem are

$$X_1(t) = \begin{bmatrix} e^{-2t} \\ e^{-2t} \end{bmatrix} \qquad \text{and} \qquad X_2(t) = \begin{bmatrix} e^{-5t} \\ -2e^{-5t} \end{bmatrix}.$$

8. The matrix

$$A = \begin{bmatrix} 4 & 3 \\ -4 & -4 \end{bmatrix}$$

has exponential

$$e^{At} = \begin{bmatrix} \frac{3}{2}e^{2t} - \frac{1}{2}e^{-2t} & -\frac{3}{4}e^{-2t} + \frac{3}{4}e^{2t} \\ e^{-2t} - e^{2t} & -\frac{1}{2}e^{2t} + \frac{3}{2}e^{-2t} \end{bmatrix}.$$

Solve the initial value problem

$$X' = AX, \quad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$