Instructions. The format of this exam is similar to the computer graded homework. You are encouraged to use scratch paper. There is limited partial credit. Only the answers you write inside the boxes will be graded.

1. Two chemicals A and B are combined to form a chemical C. The rate, or velocity, of the reaction is proportional to the product of the instantaneous amounts of A and B not converted to chemical C. Initially, there are 40 grams of A and 50 grams of B, and for each gram of B, 2 grams of A is used. It is observed that 20 grams of C is formed in 9 minutes. Round your answers to one decimal place.

(i) How much of chemical C is formed in 18 minutes?

There is $\boxed{}$ grams formed.

(ii) What is the limiting amount of C after a long time?

The limiting amount is $\boxed{}$ grams.

(iii) How much of chemicals A and B remains after a long time?

Of A there is $\boxed{}$ grams remaining.

Of B there is $\boxed{}$ grams remaining.

2. Consider the differential equation

$$y'' - y' - 20y = 0.$$

Verify that the functions e^{-4x} and e^{5x} form a fundamental set of solutions of the differential equation on the interval $(-\infty, \infty)$.

(i) The functions satisfy the differential equation. They are furthermore linearly independent since the Wronskian

$W(e^{-4x}, e^{5x}) = \boxed{}$

is not equal to zero for $x \in (-\infty, \infty)$.

(ii) Form the general solution.

$y(x) = \boxed{}$

3. The function $y_1(x) = \ln x$ is a solution of the differential equation $xy'' + y' = 0$. Use the reduction of order formula

$$y_2(x) = y_1(x) \int \frac{\exp\left(\int -P(x)dx\right)}{y_1^2(x)} dx$$

to find a second solution

$y_2(x) = \boxed{}$

4. Find the general solution of the higher-order differential equation

$$y''' - 5y'' + 3y' + 9y = 0$$

The general solution is

$y(x) =$

5. Solve the differential equation

$$y'' - 12y' + 36y = 42x + 4$$

by undetermined coefficients. The general solution is

$y(x) =$

6. Solve the differential equation

$$y'' + y = \cos^2 x$$

by variation of parameters. The general solution is

$y(x) =$

7. Solve the differential equation

$$x^2y'' + xy' + 16y = 0 \qquad \text{for} \qquad x > 0.$$

The general solution for $x > 0$ is

$y(x) =$